J Bridgewater ex dono Auctoris

THE DESCRIPTION AND VSE OF THE SECTOR,

For ſuch as are ſtudious of *Mathematicall practiſe.*

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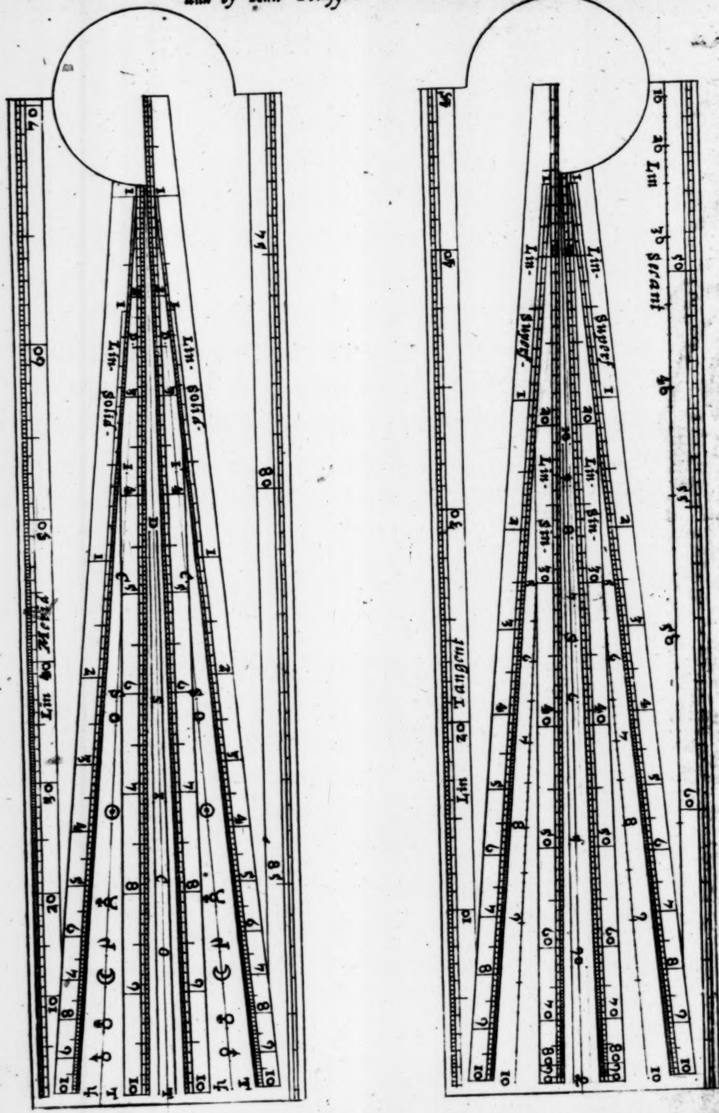
HONORATISSIMO
DOMINO *Dn. IOHANNI*
COMITI de BRIDGEWATER,
VICECOMITI de BRACKLEY,
BARONI de ELLESMERE,

HVNC SECTOREM

D. D. D.

EDM. GVNTER.

This Sector is mad by Elias Allen in Brasse
and by Iohn Tompson in woode



THE FIRST BOOKE OF THE SECTOR.

CHAP. I.

The Description, the making, and the generall vse of the Sector.

A *Sector* in Geometrie, is a figure comprehended of two right lines containing an angle at the center, and of the circumference assumed by them. This *Geometricall instrument* hauing two legs containing all varietie of angles, & the distance of the feete, representing the subtenses of the circumference, is therefore called by the same name.

It containeth 12 feuerall lines or scales, of which 7 are generall, the other 5 more particular. The first is the scale of *Lines* diuided into 100 equall parts, and numbred by 1. 2. 3. 4. 5. 6. 7. 8. 9. 10.

The second, the lines of *Superficies* diuided into 100 vnequall

vnequall parts, and numbred by 1. 1. 2. 3. 4. 5. 6. 7. 8. 9. 10.

The third, the lines of *Solids*, diuided into 1000 vnequal parts, & numbred by 1. 1. 1. 2. 3. 4. 5. 6. 7. 8. 9. 10.

The fourth, the lines of *Sines* and *Chords* diuided into 90 degrees, and numbred with 10. 20. 30. vnto 90.

These foure lines of *Lines*, of *Superficies*, of *Solids*, and of *Sines*, are all drawne from the center of the *Sector* almost to the end of the legs. They are drawne on both the legs, that euery line may haue his fellow. All of them are of one length, that they may answere one to the other. And euery one hath his parallels, that the eye may the better distinguish the diuisions. But of the parallels those onely which are inward most containe the true diuisions.

There are three other generall lines, which because they are infinite are placed on the side of the *Sector*. The first a line of *Tangents*, numbred with 10. 20. 30. 40. 50. 60. signifying so many degrees from the beginning of the line, of which 45 are equall to the whole line of *Sines*, the rest follow as the length of the *Sector* will beare.

The second, a line of *Secants*, diuided by pricks into 60 degrees, whose beginning is the same, with that of the line of *Tangents*, to which it is ioyned.

The third, is the *Meridian* line, or line of *Rumbs*, diuided vnequally into degrees, of which the first 70 are almost equall to the whole line of *Sines*, the rest follow vnto 84 according to the length of the *Sector*.

Of the particular lines inserted among the generall, because there was voyd space, the first are the lines of *Quadrature* placed betweene the lines of *Sines*, and noted with 10. 9. 8. 7. S. 6. 5. 90. *Q*.

The second, the lines of *Segments* placed betweene the lines of *Sines* and *Superficies* diuided into 50 parts, and numbered with 5. 6. 7. 8. 9. 10.

The third, the lines of *Inscribed bodies in the same Sphere*, placed betweene the scales of *Lines*, and noted with *D. S. I. C. O. T.*

The

The fourth, the lines of *Equated bodies*, placed between the lines of *Lines* and *Solids*, and noted with *D.I.C.S.O.T.*

The fift, are the lines of *Mettalls*, inserted with the lines of *Equated bodies* (there being roome sufficient) and noted with these Characters. ☉.☿.♄ ☽.♀.♂ ♃.

There remaine the edges of the *Sector*, and on the one I haue set a line of *Inches*, which are the twelth parts of a foote English : on the other a lesser line of *Tangents*, to which the *Gnomon* is *Radius*.

2 *Of the making of the Sector.*

Let a *Ruler* be first made either of brasse or of wood, like vnto the former figure, which may open and shut vpon his center. The head of it may be about the twelth part of the whole length, that it may beare the moueable foote, and yet the most part of the diuisions may fall without it. Then let a moueable *Gnomon* be set at the end of the moueable foote, and there turne vpon an *Axis*, so as it may sometime stand at a right angle with the feete, and sometimes be inclosed within the feet. But this is well knowne to the workeman.

For drawing of the lines. Vpon the center of the *Sector*, and semidiameter somewhat shorter then one of the feet, draw an occult arke of a circle, crossing the closure of the inward edges of the *Sector* about the letter *T*.

In this arke, at one degree on either side from the edge, draw right lines from the Center, fitting them with Parallels and diuide them into an hundred equall parts, with subdivisions into 2.5.or 10. as the line will beare, but let the numbers set to them, be onely 1.2.3.4.&c. vnto 10. as in the example. These lines so divided, I call the lines or scales of *Lines*; and they are the ground of all the rest.

In this Arke at 5 degrees on either side, from the edge neere *T*, drawe other right lines from the Center, and fit them with Parallells. These shall serue for the lines of *Solids*.

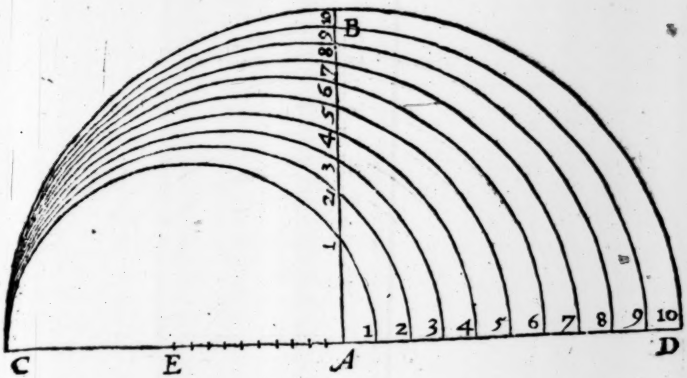
Then on the other side of the *Sector* in like manner, vpon the Center & equall Semidiameter, drawe another like Arke of a circle: & heere againe at one [redacted] degree on either side frō the edge neere the letter *Q* draw right lines from the Center, and fit them with parallells. These shall serue for the lines of *Sines.*

At 5 Degrees on either side from the edge neere *Q* drawe other right lines from the center, and fit them with parallels: these shall serue for the lines of *Superficies.*

These foure principall lines being drawne, and fitted with parallels, we may drawe other lines in the middle betweene the edges and the lines of *Lines*, which shall serue for the lines of *inscribed bodies*, and others betweene the edges and the *Sines* for the lines of *quadrature*. And so the rest as in the example.

3 *To diuide the lines of* Superficies.

SEeing like *Superficies* doe hold in the proportion of their *homologall* sides duplicated, by the 29 Pro. 6 lib. *Euclid.* If you shall find meane proportionals betweene the whole side, and each hundred part of the like side, by the 13 Pro. 6 lib. *Euclid.* all of them cutting the same line, that line so cut shal conteine the diuisions required. wherefore vpon the center *A* and Semidiamiter equall to the line of *Lines*, describe a Semicircle *A C B D*, with *A B* perpendicular to the diameter *C D*. And let the Semidiameter *A*D be diuided as the line of *Lines* into an hundred parts, & *A E* the one halfe of *AC* diuided also into an hundred parts, so shall the diuisions in *A E* be the centers from whence you shall describe the Semicircles *C* 10. *C* 20. *C* 30. &c. diuiding the lin *A B* into an hundred vnequall parts: and this line *A* B so diuided shall be the line of *Superficies*, and must be transferred into the *Sector*. But let the numbers set to them be onely 1. 1. 2. 3. vnto 10. as in the example.

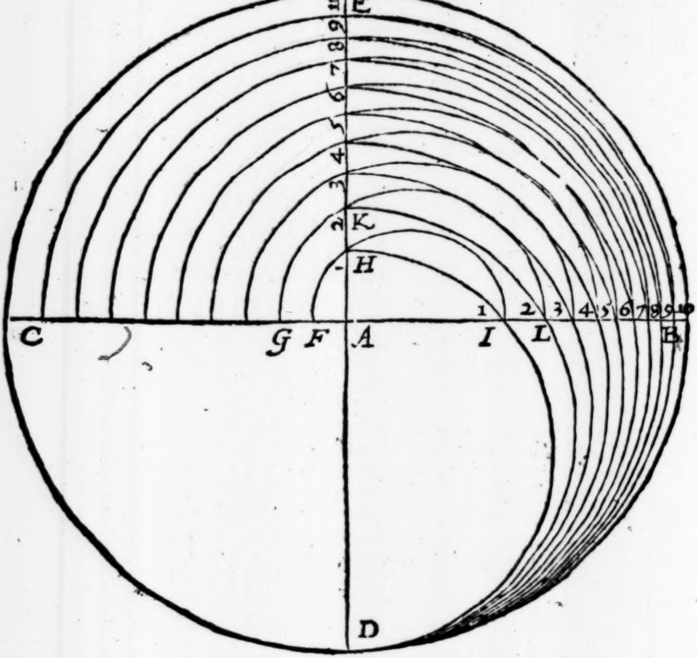


4 *To diuide the lines of* Solids.

SEing like Solids do hold in the proportion of their *homologall* ſides triplicated, if you ſhall finde two meane proportionalls between the whole ſide & each thouſand part of the like ſide : all of them cutting the ſame two right lines, the former of thoſe lines ſo cut, ſhall containe the diuiſions required.

Wherefore vpon the center *A* & Semidiameter equall to the line of *Lines*. deſcribe a circle and diuide it into 4 equall parts *C E B D*, drawing the croſſe diameters *C B*, *E D*. Then diuide the ſemidiameter *A C*, firſt into 10 equall parts, and betweene the whole line *A D* & *A F* the tenth part of *A C*, ſeeke out two meane proportionall lines *A I* and *A H*. againe betweene *A D* and *A G* being two tenth parts of *A C*, ſeeke out two meane proportionals *A L* and *A K*, and ſo forward in the reſt. So ſhall the line *A B* be diuided into 10 vnequall parts.

 Secondly



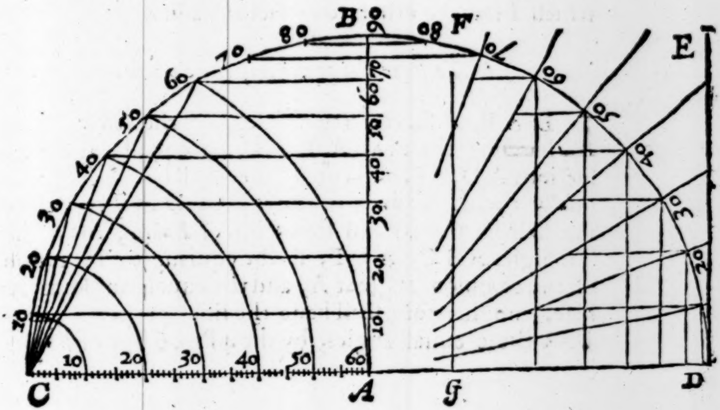
Secondly, diuide each tenth part of the line *AC* into 10 more, and betweene the whole line *AD*, and each of them, seeke out two meane proportionalls as before: So shall the line *AB* be diuided now into an hundred vnequall parts.

Thirdly, If the length will beare it, subdiuide the line *AC* once againe, each part into ten more: and betweene the whole line *AD* and each subdiuision, seeke two meane

meane proportionalls as before. So should the line *A* B be now diuided into 1000 parts. But the ruler being short, it shall suffice, if those 10 which are nearest the center be expressed, the rest be vnderstood to be so diuided, though actually they be diuided into no more then 5 or 2. and this line *A B* so diuided shall be the line of *Solids*, and must be transferred into the *Sector*: But let the numbers set to them be onely 1. 1. 1. 2. 3. &c. vnto 10. as in the example.

5 *To diuide the lines of* Sines *and* Tangents *on the side of the Sector.*

VPon the center *A*, and semidiameter equall to the line of *Lines*, describe a semicircle *A B C* D, with *A B*, perpendicular to the diameter *C* D. Then diuide the quadrants *C B*, *B* D, each of them into 90. and subdiuide each degree into 2 parts: For so, if streight lines be drawne parallell to the diameter *C* D, through these 90, and their subdiuisions they shall diuide the perpendicular *A* B vnequally into 90.



And this line A B so diuided shall be the line of *Sines*; and must be transferred into the *Sector*. The numbers set to them are to be 10. 20. 30. &c. vnto 90 as in the example.

If now in the poynt *D*, vnto the diameter *C D*, we shall raise a perpendicular *D E*, and to it drawe streight lines from the center *A*, through each degree of the quadrant DB. This perpendicular so diuided by them shall be the line of *Tangents*, & must be transferred vnto the side of the *Sector*. The numbers set to them, are to be 10. 20. 30. &c. as in the example.

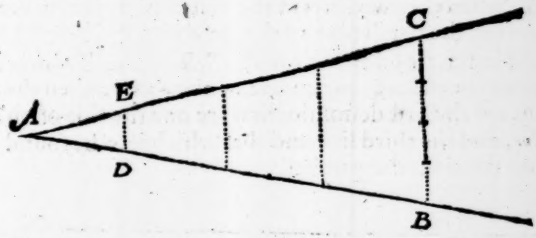
If betweene A and D, another streight line *G F*, be drawne parallell to D E, it will be diuided by those lines from the center in like sort as D E is diuided, and it may serue for a lesser line of *Tangents*, to be set on the edge of the *Sector*.

These lines of *Sines* and *Tangents*, may yet otherwise be transferred into the *Sector* out of the line of *Lines*, (or rather out of a diagonall Scale equall to the line of *Lines*) by tables of Sines and Tangents. In like manner may the lines of *Superficies*, be transferred by tables of square rootes; and the line of *Solids*, by tables of cubique rootes: which I leaue to others to extract at leasure.

6 *To shew the ground of the Sector.*

Let A B, A C, represent the leggs of the *Sector*: then seeing these two A B, A C, are equall, and their sections A D, AE, also equall, they shall be cut proportionally: and if we draw the lines B C, D E, they will be parallell by the second Pro. 6 lib. of *Euclid*, and so the Triangles A B C, A D E, shalbe equiangle; by reason of the common angle at A, and the equall angles at the base, and therefore shall haue the sides proportionall about those equall angles, by the 4 Pro. 6 lib. of *Euclid*.

The



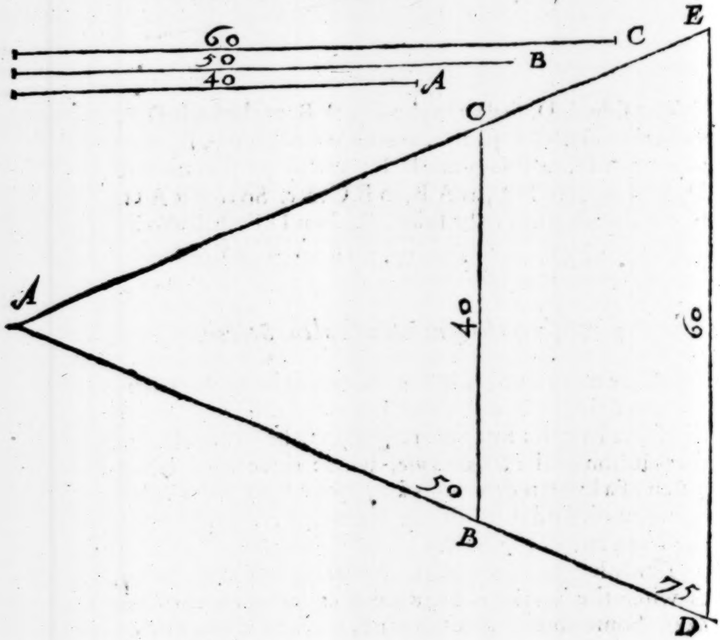
The ſide A D, ſhalbe to the ſide A B, as the baſis D E, vnto the parallell baſis BC, and by conuerſion A B, ſhall be vnto A D, as B C, vnto D E: and by permutation A D, ſhall be vnto D E, as A B, to B C.&c. So that if A D, be the fourth part of the ſide A B, then D E, ſhall alſo be the fourth part of his parallell baſis B C. The like reaſon holdeth in all other ſections.

7 *To ſhew the generall vſe of the Sector.*

THere may ſome cõcluſions be wrought by the *Sector*, euen then when it is ſhut, by reaſon that the lines are all of one length: but generally the vſe hereof conſiſts in the ſolution of the *Golden rule*, where three lines being giuen of a known denomination, a fourth proportionall is to be found. And this ſolution is diuerſe in regard both of the *lines*, and of the *entrance* into the worke.

The ſolution in regard of the *lines* is ſometimes *ſimple*, as when the worke is begun and ended vpon the ſame *lines*. Sometimes it is compound, as when it is begun on one kind of *lines*, and ended on another. It may be begun vpon the lines of *Lines*; & finiſhed vpon the lines of *Superficies*. It may begin on the *Sines*, and end on the *Tangents*.

The ſolution in regard of the ***entrance*** into the worke, may be either with a ***parallell*** or elſe ***laterall*** on the ſide of the Sector, I cal it ***parallell entrance***, or entring with a parallell, when the two lines of the firſt denomination are applied in the parallells, and the third line, and that which is ſought for, are on the ſide of the *Sector*. I call it ***laterall entrance***, or entring on the ſide of the ***Sector***, when the two lines of the firſt denomination are one the ſide of the ***Sector***, and the third line and that which is to be found out, doe ſtand in the parallells.

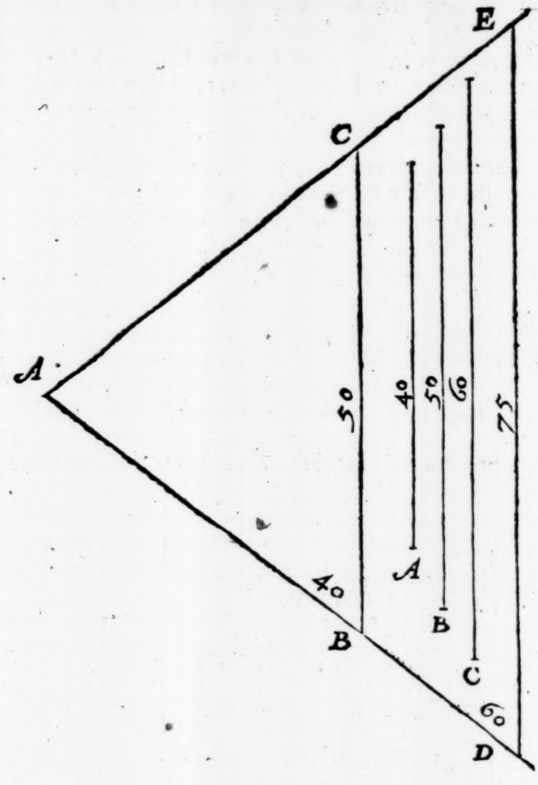


As for example, let there be giuen three lines A,B,C, to which I am to find a fourth proportionall. let A, measured in the line of *lines*, be 40, B 50, and C 60, and suppose the question be this. If 40 *Monthes* giue 50 *pounds*, what shal 60? Here are lines of two denominatiõs, one of *months*, another of *pounds*, and the first with which I am to enter must be that of 40 *monthes*. If then I would *enter with a parallell*, first I take A, the line of 40, and put it ouer as a *parallell* in 50, reckoned in the line of *lines*, on either side of the *Sector* from the center, so as it may be the Base of an *Isoscheles* triangle B A C, whose sides A B, A C are equal to B, the line of the second denomination.

Then the *Sector* being thus opened, I take C the line of 60, betweene the feete of the compasses, and carrying them *parallell* to B C, I finde them to crosse the lines A B, A C, on the side of the *Sector* in D and E, numbred with 75, wherefore I conclude the line A D, or A E, is the fourth proportionall and the correspondent number 75 which was required.

But if I would *enter on the side* of the *Sector*, then would I dispose the lines of the first denomination A and C, in the line of *Lines*, on both sides of the *Sector*, in A B, A C, & in A D, A E, so as they should all meete in the center A, and then taking C the line of the second denomination put it ouer as a *parallell* in B C, that it may be the Basis of the *Isoscheles* triangle B A C, whose sides A B, A C, are equall to A, the first line of the first denomination, for so the *Sector* being thus opened, the other *parallell* from D to E, shall be the fourth proportionall which was required, and if it be measured with the other lines, it shal be 75, as before.

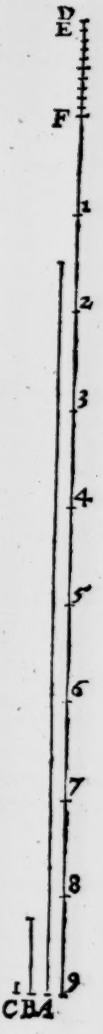
In both these manner of operations, the two first lines do serue to opẽ the *Sector* to his due angle, the difference betweene them is especially this, that in *parallell entrance*, the two lines of the first denomination, are placed in the parallells B C, D E, & in *latterall entrance* they are placed on both sides of the *Sector*, in A B, A D and in A C, A E.



Now in *simple solution* which is begun and ended, vpon the same kinde of lines, it is allone which of the two latter lines be put in the secōd or third places. As in our exāple we may say, as 40 are to 50, so 60 vnto 75, or else as 40 are to 60, so 50 vnto 75. And hence it cōmeth that we may enter both with a *parrallell*, & on the sides two manner of wayes at either entrance, and so the most part of questions

may

may be wrought 4 ſeuerall wayes, though in the propoſitions following, I mention onely that which is moſt conuenient. Thus much for the generall vſe of the *Sector*, which being conſidered and well vnderſtood, there is nothing hard in that which followeth.



CHAP. II.

The vſe of the Scale of Lines.

1. *To ſet downe a Line, reſembling any giuen parts or fraction of parts.*

THe lines of *Lines* are diuided actually into 100 parts, but we haue put onely 10 numbers to them. Theſe we would haue to ſignifie either themſelues alone, or ten times themſelues, or an hundred times themſelues, or a thouſand times themſelues, as the matter ſhall require. As if the numbers giuen be no more then 10, then we may thinke the lines onely diuided into 10 parts according to the numbers ſet to them. If they be more then 10, and not more then 100, then either line ſhall containe 100 parts, and the numbers ſet by them ſhall be in value 10. 20. 30. &c. as they are diuided actually. If yet they be more then 100, then euery part muſt be thought to be diuided into 10, and either line ſhall be 1000 parts, and the numbers ſet to them ſhall be in value 100. 200. 300, and ſo forward ſtill increaſing themſelues by 10. This being preſuppoſed, we may number the parts and fraction of parts giuen in the line of *lines*; and taking out the diſtance with a paire of compaſſes, ſet it by, for the line ſo taken ſhall reſemble the number giuen.

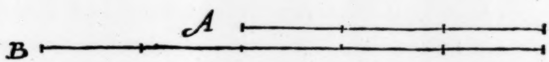
In this manner may we ſet downe a line reſembling 75, if either we take 75 out of the hundred parts, into which one of the line of *lines* is actually diuided, and note it in A, or $7\frac{1}{2}$. of the firſt 10 parts, and note it in B, or onely $\frac{3}{4}$. of one of thoſe hundred parts, and note it in C. Or

if this be either to great or to ſmall, we may run a Scale at pleaſure, by opening the compaſſe to ſome ſmall diſtance, and running it ten times ouer, then opening the compaſſe to theſe ten, run them ouer nine times more, & ſet figures to them as in this example, and out of this we may take what parts we will as before.

To this end I haue diuided the line of inches on the edge of the *Sector*, ſo as one inch containeth 8 parts, another 9, another 10, &c. according as they are figured, and as they are diſtant from the other end of the *Sector*, that ſo we might haue the better eſtimate.

2 To encreaſe a line in a giuen proportion.
3 To diminiſh a line in a giuen proportion.

TAke the line giuen with a paire of compaſſes, and open the *Sector*, ſo as the feete of the compaſſes may ſtand in the points of the number giuen, then keeping the *Sector* at this angle, the parallell diſtance of the points of the number required, ſhall giue the line required.



Let *A*, be a line giuen to be increaſed in the proportion of 3 to 5. Firſt I take the line *A*, with the compaſſes, and open the *Sector* till I may put it ouer in the poynts of 3 and 3, ſo the parallell betweene the poynts of 5 & 5, doth giue me the line *B*, which was required.

In like manner, if *B*, be a line giuen to be diminiſhed in the proportiō of 5 to 3, I take the line *B* & to it open the *Sector* in the poynts of 5, ſo the parallell betweene the points of 3, doth giue me the line *A*, which was required.

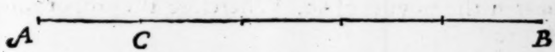
If this manner of worke doth not ſuffice, we may multiplie or diuide the numbers giuen by 1, or 2, or 3, &c. And ſo worke by their numbers *equimultiplices*, as for 3

and

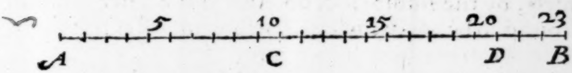
and 5, wee may open the *Sector* in 6 and 10, or else in 9 and 15, or else in 12 and 20, or in 15 and 25, or in 18 and 30. &c.

4 *To diuide a line into parts giuen.*

TAke the line giuen, and open the *Sector* according to the length of the said line in the points of the parts, whereunto the line should be diuided, then keeping the *Sector* at this angle, the parallell distance betweene the points of 1 and 1 shall diuide the line giuen into the parts required.



Let A B, be the line giuen to be diuided into fiue parts, first I take this line A B, and to it open the *Sector* in the points of 5 and 5, so the parallell betweene the points of 1 and 1, doth giue me the line A C, which doth diuide it into the parts required.



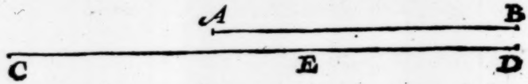
Or let the like line A B, be to be diuided into twenty three parts. First I take out the line and put it vpon the *Sector* in the points of 23, then may I by the former proposition diminish it in A C, C D, in the proportion of 23, to 10, and after that diuide the line A C into 10, &c. As before.

5 *To finde a proportion betweene two or more right lines giuen.*

TAke the greater line giuen, and according to it open

the

the *Sector* in the points of 100 and 100, then take the lesser lines seuerally, & carry them parallell to the greater, till they stay in like points, so the number of points wherein they stay, shall shew their proportion vnto 100.



Let the lines giuen be *AB*, *CD*, first I take the line *CD*, & to it open the *Sector* in the points of 100, and 100, then keeping the *Sector* at this angle, I enter the lesser line *AB*, parallell to the former, and find it to crosse the lines of *Lines* in the poynts of 60. Wherefore the proportion of *A B* to *C D*, is as 60 to 100.

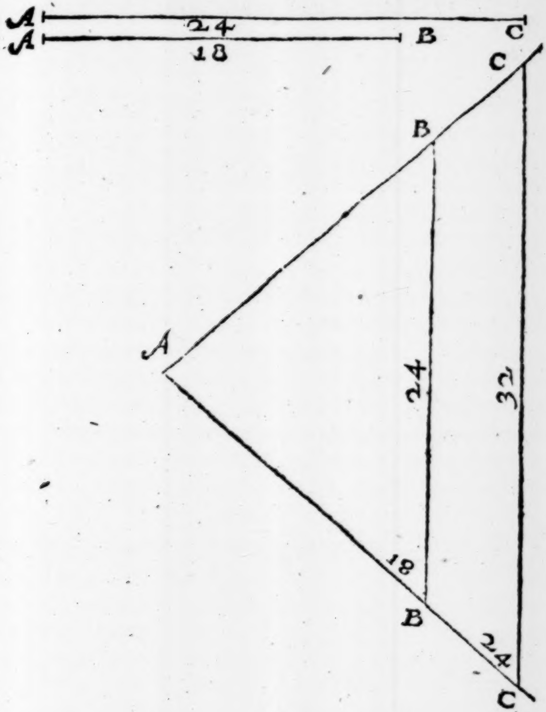
Or if the line *C* D, be greater then can be put ouer in the poynts of 100, then I admit the lesser line *A B*, to be 100, & cutting off *CE* equal to *A B*, I find the proportion of *C E*, vnto *E* D, to be as 100, almost to 67; wherefore this way ẏ proportiō of *AB* vnto CD, is as 100 vnto almost 167.

This proposition may also not vnfitly be wrought by any other number, that admits seuerall diuisions, and namely, by the numbers of 60. And so the lesser line will be found to be 36, which is as before in lesser numbers, as 3 vnto 5. It may also be wrought without opening the *Sector*. For if the lines between which we seek a proportion, be applyed to the lines of *Lines*, (or any other Scale of equall parts) there will be such proportiō found between them, as betweene the lines to which they are equall.

6 *Two lines being giuen to finde a third in continuall proportion.*

First place both the lines giuen, on both sides of the *Sector* from the Center, and marke the termes of their extension, then take out the second line againe, and to it open the *Sector*, in the termes of the first line, so keeping the

the *Sector* at this angle, the parallell distance betweene the termes of the second line, shall be the third proportionall.

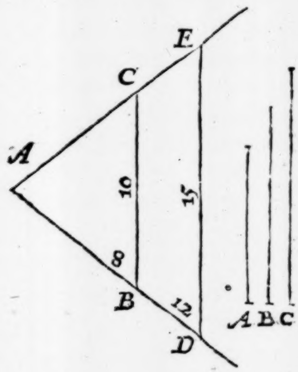


Let the two lines giuen be A B, A C, which I take out and place on both sides of the *Sector*, so as they all meete in the center A, let the termes of the first line be B and B, the termes of the second C and C. Then doe I take out A C the second line againe, and to it open the *Sector* in the termes B B. So the parallell betweene C and C doth giue me the third line in continuall proportion. For as *AB* is vnto *AC*, so BB, equall to *AC*, is vnto *C C*.

7 *Three lines being giuen to finde the fourth in discontinuall proportion.*

Here the first line and the third are to be placed on both sides of the *Sector* from the center, then take out the second line, and to it open the *Sector* in the termes of the first line. For so keeping the *Sector* at this angle, the parallell distance between the termes of the third line, shalbe the fourth proportionall.

Let the three lines giuen be A, B, C.



First I take out A and C, and place them on both sides of the *Sector*, in A B, A C, and A D, A E, laying the beginning of both lines at the center A, then do I take our B the second line, according to it I open the *Sector* in B and C, the termes of the first line: so the parallell betweene D and E, doth giue me the fourth proportionall which was required.

As in *Arithmetique*, it sufficeth if the first and third number giuen be of one denomination, the second & the fourth which is required be of another. For one and the same denomination is not required necessarily in them all. So in *Geometrie*, it sufficeth if the sides A B, A D, resembling the first

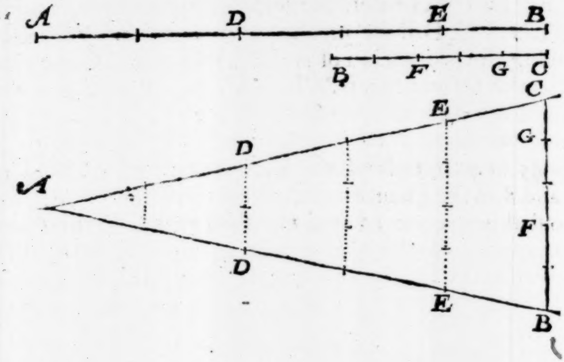
first and third lines giuen be measured in one Scale, and the parallells B C, D E be measured in another. Wherefore knowing the proportion of A the first line, and C the third line, by the fift *prop.* before. Which is here as 8 to 12, & descē-ding in lesser numbers is as 4 to 6, or as 2 to 3, or ascending in greater numbers, as 16 vnto 24, or 18 to 27, or 20 to 30, or 30 to 45, or 40 to 60 &c. If the *Sector* be opened in the points of 8 and 8, to the quantity of *B*, the second line giuen, then a parallell betweene 12 and 12, shall giue D E, the fourth line required. So likewise if it be opened in 4 and 4, then a parallell betweene 6 and 6, or if in 16 and 16, then a parallell betweene 24 and 24 shall giue the same D E. And so in the rest.

8 *To diuide a line in such sort as another line is before diuided.*

FIrst take out the line giuen, which is already diuided, and laying it on both sides of the *Sector* from the center, marke how farre it extendeth. Then take out the second line which is to be diuided, and to it open the *Sector* in the termes of the first line. This done, take out the parts of the first line, and place them also on the same side of the *Sector* from the center. For the parallells taken in the termes of these parts, shalbe the correspondent parts in the line which is to be diuided.

Let *A B*, be a line diuided in *D* and *E*, and *B C*, the line which I am to diuide in such sort, as *A B* is diuided.

First I take out the line *A B*, and place it on the line of *Lines* in *A B*, *A C*, both from the center *A*, then take I out the second *B C*, and to it open the *Sector* in *B* and *C*, the termes of the first line. The *Sector* thus opened to his due angle, I take out *A D* and *A E*, the parts of the first line *A B*, and place them also on both the sides of the *Sector* in *A D*, *A E*, so the parallell *D D*, giueth me *B F*, and the parallell *E E*, giueth me *B G*, and now the line *B C*, is diuided in *F* & *G*, as is the other line *A B*, in D and *E*, which is that which was



required.

If the line *AB*, were longer then one of the sides of the Ruler, then should I finde what proportion it hath to his parts *A* D, *A E*, and that knowne I may worke as before in the former proposition.

9 *Two numbers being giuen to finde a third in continuall proportion.*

FIrst reckon the two numbers giuen on both sides of the lines of *Lines* from the center, and marke the termes to which either of them extendeth, then take out a line resembling the second number againe, and to it open the *Sector* in the termes of the first number, for so keeping the *Sector* at this angle, the parallell distance betweene the termes of the second laterall number, being measured in the same Scale, from whence his parallell was taken, shall giue the third number proportionall.

Let the two numbers giuen be 18, 24, these being resembled in lines, the worke will be in a manner all one, with that in the sixt *Prop.* and so the third proportionall number will be found to to be 32.

10 *Three*

10. *Three numbers being giuen to finde a fourth in discontinuall proportion.*

THe solution of this proposition, is in a manner all one with that before in the seuenth *Prop.* onely there may be some difficulty in placing of the numbers. To avoyd this, we must remember that three numbers being giuen, the question is annexed but to one, and this must allwayes be placed in the third place, that which agrees with this third number in denomination, shalbe the first number, and that which remaineth the second number. This being considered, reckon the first, and third numbers, which are of the first denomination on both sides of the lines of *Lines* from the center, and marke the termes to which either of them extendeth, then take out a line resembling the second number, and to it open the *Sector* in the termes of the first number, for so keeping the *Sector* at this angle, the parallell distance betweene the termes of the third laterall number, being measured in the same Scale from whence his parallell was taken, shall giue the fourth number proportionall.

As if a question were proposed in this manner, 10 yards cost 8 *l*, how many yards may we buy for 12 *l*? heere the question is annexed to 12; and therefore it shall be the third number, and because 8 is of the same denomination, it shall be the first number, then 10 remaining, it must be the second number, so will they stand in this order, 8, 10, 12. These being resembled in lines, the worke will be in a manner the same, with that in the seuenth *Prop.* and the fourth proportionall number will be found to be 15. For as 8 are to 10, so 12 vnto 15.

And this holdeth in direct proportion, where, as the first number is to the second, so the third to the fourth. So that if the third number be greater then the first, the fourth will be greater then the second, or if the third number be lesse then the first, the fourth will be lesse then the second, but in *reciprocall* proportion, commonly called the *Back rule,*

 where

where by how much the firſt number is greater then the third, ſo much the ſecond will be leſſe then the fourth, or by how much the firſt number is leſſe then the third, ſo much the ſecond will be greater then the fourth. The manner of working muſt be contrary, that is ; the *Sector* is to be opened in the termes of the third number, and the parallell reſembling the number required, is to be found betweene the termes of the firſt number, the reſt may be obſerued as before, as for example.

If twelue men would raiſe a frame in ten dayes, in how many dayes would eight men raiſe the ſame frame ? Here, becauſe the fewer men would require the longer time, though the numbers be 12, 10, 8, *yet the fourth proportionall will be found to be* 15.

75. *So if* 60 *yards, of three quarters of a yard in bredth, would hang round about a roome, and it were required to know how many*
50. *yards of halfe a yard in bredth, would ſerue for the ſame roome. The fourth proportionall would be found to be* 90.

So if to make a foote ſuperficiall, 12 *inches in bredth doe require* 12 *inches in length, and the bredth being* 16 *inches, it were required to knowe the length. Here, becauſe the more bredth, the leſſe length, the fourth proportionall will be found to be* 9.

So if to make a Solid foote, a baſe of 144 *inches, require* 12 *inches in height, and a baſe giuen being* 216 *inches, it were required to knowe how many inches it ſhall haue in height. The fourth proportionall would be found to be* 8.

This laſt propoſition of finding a fourth proportionall number, may be wrought alſo by the lines of *Superficies*, and by the lines of *Solids*.

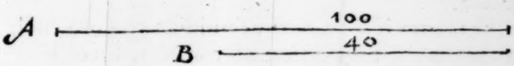
CHAP. III.

CHAP, III.

The vſe of the lines of Superficies.

1 *To finde a proportion betweene two or more like* Superficies.

TAke one of the ſides of the greater *Superficies* giuen, and according to it open the *Sector* in the points of 100 and 100, in the lines of *Superficies*, then take the like ſides of the leſſer *Superficies* ſeuerally, and carry them parallell to the former, till they ſtay in like points, ſo the number of points wherein they ſtay, ſhall ſhew their proportion vnto 100.

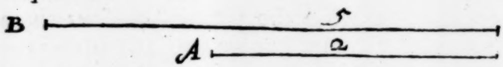


Let *A* and *B*, be the ſides of like *Superficies*, as the ſides of two ſquares, or the diameters of two circles, firſt I take the ſide *A*, and to it open the *Sector* in the points of 100, then keeping the *Sector* to this angle, I enter the leſſer ſide *B*, parallell to the former, and finde it to croſſe the lines of *Superficies* in the points of 40, wherefore the proportion of the *Superficies*, whoſe ſide is *A*, to that whoſe ſide is *B*, is as 100 vnto 40, which is in leſſer numbers, as 5 vnto 2.

This propoſition might haue beene wrought by 60, or any other number that admits ſeuerall diuiſions. It may alſo be wrought without opening the *Sector*, for if the ſides of the *Superficies* giuen, be applied to the lines of *Superficies* beginning alwayes at the center of the *Sector*, there will be ſuch proportion found betweene them, as betweene the number of parts whereon they fall.

2 *To augment a Superficies in a giuen Proportion.*
3 *To diminiſh a Superficies in a giuen Proportion.*

TAke the ſide of the *Superficies*, and to it open the *Sector* in the points of the numbers giuen; then keeping the *Sector* at that angle, the parallell diſtance between the points of the number required, ſhall giue the like ſide of the *Superficies* required.



Let *A* be the ſide of a Square to be augmented in the proportion of 2 to 5. Firſt I take the ſide *A*, and put it ouer in the lines of *Superficies*, in 2 and 2; ſo the parallel between 5 and 5, doth giue me the ſide *B*, on which if I ſhould make a Square, it would haue ſuch proportion to the ſquare of *A*, as 5 vnto 2.

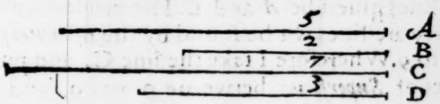
In like maner if *B* were the ſemidiameter of a circle to be diminiſhed in the proportion of 5 vnto 2, I would take out *B*, and put it ouer in the lines of *Superficies*, in 5 and 5; ſo the parallell betweene 2 and 2, would giue me *A*; on which Semidiameter if I ſhould make a circle, it would be leſſe then the circle made vpon the Semidiameter *B*, in ſuch proportion as 2 is leſſe then 5.

For varietie of worke the like caution may be here obſerued to that which we gaue in the third *Prop.* of *Lines.*

4 *To adde one like Superficies to another.*
5 *To ſubtract one like Superficies from another.*

FIrſt, the proportion betweene like ſides of the *Superficies* giuen, is to be found by the firſt *Prop.* of *Superficies*, then adde or ſubtract the numbers of thoſe proportions, and accordingly augment or diminiſh by the former *Prop.*

As

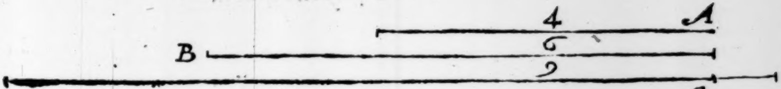


As if *A* and *B* were the sides of two Squares, and it were required to make a third Square equall to them both. First the proportion betweene the squares of *A* and *B*, would be found to be as 100 vnto 40, or in the lesser numbers as 5 to 2; then because 5 and 2 added do make 7, I augment the side *A* in the proportion of 5 to 7, and produce the side *C*, on which if I make a square, it will be equall to both the squares of *A* and *B*, which was required.

In like maner *A* and *B* being the sides of two Squares, if it were required to subtract the square of *B* out of the square of *A*, and to make a square equall to the remainder, here the proportion being as 5 to 2, because 2 taken out of 5, the remainder is 3, I would diminish the side *A* in the proportion of 5 to 3, and so I should produce the side *D*, on which if I make a square, it will be equall to the remainder when the square of *B* is taken out of the square of *A*, that is, the two squares made vpon *B* and *D*, shall be equall to the first square made vpon the side *A*.

6 *To find a meane proportionall betweene two lines giuen.*

First find what proportion is betweene the lines giuen, as they are lines, by the fifth *Prop.* of *Lines*, then open the *Sector* in the lines of *Superficies*, according to his number, to the quantitie of the one, and a parallell taken betweene the points of the number belonging to the other line shall be the meane proportionall.



Let the lines giuen be *A* and *C*. The proportion between them as they are lines wil be found by the fifth *Prop.* of *Lines* to be as 4 to 9. Wherefore I take the line C, and put it ouer in the lines of *Superficies* betweene 9 and 9, and keeping the *Sector* at this angle, his parallell betweene 4 and 4 doth giue me *B* for the meane proportionall. Then for proofe of the operation I may take this line B, and put it ouer between 9 and 9: ſo his parallel between 4 and 4, ſhall giue me the firſt line A. Whereby it is plaine that theſe three lines do hold in continuall proportion; and therefore B is a meane proportionall betweene A and C the extremes giuen.

Vpon the finding out of this meane proportion depend many Corollaries, as

To make a Square equall to a Superficies giuen.

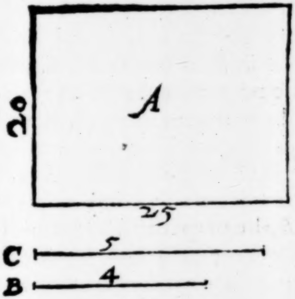
IF the *Superficies* giuen be a rectangle parallellogram, a meane proportionall betweene the two vnequal ſides ſhall be the ſide of his equall ſquare.

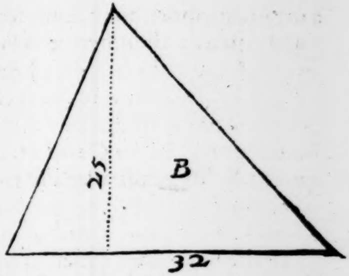
If it ſhall be a triangle, a meane proportion betweene the perpendicular and halfe the baſe ſhal be the ſide of his equal ſquare. If it ſhall be any other right-lined figure, it may be reſolued into triangles, and ſo a ſide of a ſquare found equall to euery triangle; and theſe being reduced into one equall ſquare, it ſhall be equall to the whole right-lined figure giuen.

To finde a proportion betweene Superficies, though they be vnlike one to the other.

IF to euery *Superficies* we find the ſide of his equall ſquare, the proportion betweene theſe ſquares, ſhall be the proportion betweene the *Superficies* giuen.

Let

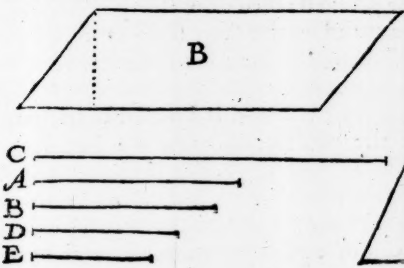


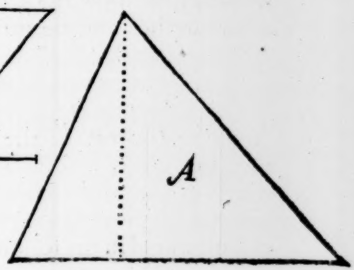


Let the *Superficies* giuen, be the oblonge *A.* and the triangle *B.* First between the vnequall sides of *A*, I find a meane proportionall, and note it in *C:* this is the side of a square equall vnto *A.* Then betweene the prependicular of B, and halfe his base, I finde a meane proportionall, and note it in B: this is the side of a Square equall to *B:* but the proportion betweén the squares of *C* and B, will be found by the first *Prop.* of *Superficies* to be as 5 to 4: and therefore this is the proportion betwene those giuen *Superficies.*

To make a Superficies like to one Superficies and equall to another.

LEt the one *Superficies* giuen be the *triangle* A, and the other the *Rhomboides* B; and let it be required to make an-





other *Rhomboides* like to B, and equall to the *triangle* A.

First between the perpendicular and the base of *B*, I find a meane proportionall, and note it in *B*, as the side of his equall square: then betweene the perpendicular of the triangle *A*, and halfe his base, I find a meane proportionall, and note it in *A*, as the side of his equall square. Wherefore now as the side *B* is to the side *A*, so shall the sides of the Rhomboides giuen be to *C* and *D*, the sides of the Romboides required, & his pendicular also to *E*, the perpēdicular required.

perpend =

Hauing the sides and the perpendicular, I may frame the *Rhomboides* vp, and it will be equall to the triangle *A*.

If the *Superficies* giuen had been any other right-lined figures, they might haue been resolued into triangles, and then brought into squares as before.

Many such Corollaries might haue been annexed, but the meanes of finding a meane proportionall being knowne, they all follow of themselues.

7 *To finde a meane proportionall betweene two numbers giuen.*

FIrst reckon the two numbers giuen on both sides of the Lines of *Superficies*, from the center, and mark the termes whereunto they extend; then take a line out of the Line of *Lines*, or any other scale of equall parts resembling one of those numbers giuen, and put it ouer in the termes of his like number in the lines of *Superficies*; for so keeping the *Sector* at this angle, the parallell taken from the termes of the other number and measured in the same scale from which the other parallell was taken, shall here shew the meane proportionall which was required.

Let the numbers giuen be 4 and 9. If I shall take the line *A*, in the *Diagram* of the sixt *Prop.* resembling 4 in a scale of equall parts, and to it open the *Sector* in the termes of 4 and 4, in the lines of *Superficies*, his parallell betweene 9 and 9 doth giue me *B* for the meane proportionall. And this measured in the scale of equall parts doth extend to 6, which

which is the meane proportionall number between 4 and 9.

For as 4 to 6, ſo 6 to 9.

In like maner if I take the line *C*, reſembling 9 in a ſcale of equall parts, and to it open the *Sector* in the termes of 9 and 9, in the lines of *Superficies*, his parallell between 4 and 4 doth giue me the ſame line *B*, which will proue to be 6, as before, if it be meaſured in the ſame ſcale whence *D* was taken.

8 *To find the ſquare roote of a number.*
9 *The roote being giuen to find the ſquare number of that roote.*

IN the extraction of a ſquare roote it is vſuall to ſet pricks vnder the firſt figure, the third, the fifth, the ſeuenth, and ſo forward, beginning from the right hand toward the left, and as many pricks as fall to be vnder the ſquare number giuen, ſo many figures ſhall be in the roote: ſo that if the number giuen be leſſe then 100, the roote ſhall be only of one figure; if leſſe then 10000, it ſhall be but two figures; if leſſe then 1000000, it ſhall be three figures, &c.

Thereupon the lines of *Superficies* are diuided firſt into an hundred parts, and if the number giuen be greater then 100, the firſt diuiſion (which before did ſignifie only one) muſt ſignifie 100, and the whole line ſhall be 10000 parts: if yet the number giuen be greater then 10000, the firſt diuiſion muſt now ſignifie 10000, and the whole line be eſteemed at 1000000 parts: and if this be too little to expreſſe the number giuen, as oft as we haue recourſe to the beginning, the whole line ſhall increaſe it ſelfe an hundred times.

By this meanes if the laſt pricke to the left hand ſhall fall vnder the laſt figure, which will be as oft as there be odde figures, the number giuen ſhall fall out betweene the center of the *Sector* and the tenth diuiſion: but if the laſt prick ſhall fall vnder the laſt figure but one, which will be as oft as there be euen figures, then the number giuen ſhall fall out betweene the tenth diuiſion and the end of the *Sector*.

This being considered, when a number is giuen and the square roote is required, take a paire of compasses and setting one foote in the center, extend the other to the terme of the number giuen in one of the lines of *Superficies*; for this distance applied to one of the Lines of *Lines*, shall shew what the Square roote is, without opening the *Sector*.

Thus 64 doth giue a roote of 8, and 860 a roote of almost 19, and 1296 a roote of 36, and 7056 a roote of 84, and 62500 a roote of 250, and 714000 a roote of about 845, and so in the rest. 360

On the contrary, a number giuen may be squared, if first we extend the compasses to the number giuen in the lines of *Lines*, and then apply the distance to the *Lines* of *Superficies*, as may appeare by the former examples.

10 *Three numbers being giuen to find the fourth in a duplicated proportion.*

It is plaine by the 19 and 20 *Prop.6.Lib.* of *Euclid*, that like *Superficies* do hold in a duplicated proportion of their homologall sides, whereupon a qnestion being moued concerning *Superficies* and their sides. It is vsuall in Arithmeticke that the proportion be first duplicated before the question be resolued, which is not necessarie in the vse of the *Sector*, only the numbers which do signifie *Superficies* must be reckoned in the lines of *Superficies*, and they which signifie the sides of *Superficies*, in the lines of *Lines*, after this maner.

If a question be made concerning a *Superficies*, the two numbers of the first denomination must be reckoned in the lines of *Lines*, and the *Sector* opened in the termes of the first number to the quantitie of a line out of the scale of *Superficies* resembling the second number; so his parallels taken betweene the termes of the third number, being measured in the same scale of *Superficies*, shall giue the Superficiall number which was required.

As if a Square, whose side is fortie perches in length, shall con-

containe ten acres in the *Superficies*, and it be required to know how many acres the Square ſhould contain, whoſe ſide is ſixtie perches.

Here if I tooke 10 out of the line of *Superficies*, and put it ouer in 40 in the lines of *Lines*, his parallell between 60 and 60 meaſured in the line of *Superficies*, would be $22\frac{1}{2}$; and ſuch is the number of acres required. For Squares do hold in a duplicated proportion of their ſides; wherefore when the proportion of their ſides is as 4 to 6, and 4 multiplied into 4 become 16, and 6 multiplied into 6 become 36, the proportion of their ſquares ſhall be as 16 to 36; and ſuch is the proportion of 10 to $22\frac{1}{2}$.

If a field meaſured with a ſtatute perch of $16\frac{1}{2}$ foote, ſhall containe 288 acres, and it be required to know how many acres it would containe if it were meaſured with a woodland perch of 18 foote.

Here becauſe the proportion is reciprocall, if I tooke 288 out of the line of *Superficies*, and put it ouer in 18, in the lines of *Lines*, his parallell betweene $16\frac{1}{2}$ and $16\frac{1}{2}$ meaſured in the line of *Superficies*, would be 242; and ſuch is the number of acres required.

For ſeeing the proportion of the ſides is as $16\frac{1}{2}$ to 18, or in leſſer numbers as 11 to 12, and that 11 multiplied into 11 become 121, and 12 into 12 become 144, the proportion of theſe *Superficies* ſhall be as a 121 to 144, and ſo haue 288 to 242, in *reciprocall* proportion.

On the contrary, if a queſtion be propoſed concerning the ſide of a *Superficies*, the two numbers of the firſt denomination muſt be reckoned in the lines of *Superficies*, and the *Sector* opened in the termes of the firſt number, to the quantitie of a line, out of the line of *Lines*, or ſome Scale of equall parts, reſembling the ſecond number; ſo his parallell taken between the termes of the third number being meaſured in the ſame ſcale with the ſecond number, ſhal giue the fourth number required.

As if a field contained 288 acres when it was meaſured with a ſtatute perch of $16\frac{1}{2}$, and being meaſured with ano-

ther

ther perch, was found to containe 242 acres, it were required to know what was the length of the perch with which it was so measured.

Here because the proportion is reciprocall, if I tooke $16\frac{1}{2}$ out of the line of *Lines*, and put it ouer in 242 in the lines of *Superficies*, his parallell betweene 288 and 288, being measured in the line of *Lines*, would be 18, and such is the length of the perch in foote whereby the field was last measured.

For seeing the proportion of the acres is as 288 vnto 242, or in the least numbers as 144 to 121, and that the roote of 144 is 12, and the root of 121 is 11, the proportion of roots and consequently of the perches shall be as 12 to 11, and so are $16\frac{1}{2}$ to 18, in *reciprocall* proportion.

If 360 men were to be set in forme of a long square, whose sides shall haue the proportion of 5 to 8; and it were required to know the number of men to be placed in front and file: if the sides were onely 5 and 8, there should be but 40 men; but there are 360: therefore working as before, I find that

As 40 to the square of 5,
so 360 to the square of 15.

As 40 to the square of 8,
so 360 to the square of 24.

and so 15 and 24 are the sides required.

If 1000 men were lodged in a square ground, whose side were 60 paces, and it were required to know the side of the square wherein 5000 might be so lodged, here working as before, I should find that

As 1000 are to the square of 60:
so 5000 to the square of 134.

And such very neare is the number of paces required.

CHAP.

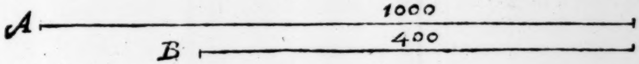
CHAP. IV.

The vse of the lines of Solids.

To finde a proportion betweene two or more like Solids.

IN the Sphere, in regular, parallell, and other like bodies, whose sides next the equall angles are proportionall, the worke is in a manner the same, with that in the first *Prop.* of *Superficies*, but that it is wrought on other lines.

Take one of the sides of the greater *Solid*, & according to it open the *Sector* in the points of 1000 and 1000, in the lines of *Solids*, then take the like sides of the lesser *Solids* seuerally, and carry them parallell to the former, till they stay in like points, so the number of points wherein they stay, shall shew their proportion to 1000.



Let *A* and *B*, be the like sides of like Solids, either the diameters, or semidiameters of two spheres, or the sides of two cubes, or other like. First I take the side *A*, and to it open the *Sector* in the points of 1000, then keeping the *Sector* at this angle, I enter the lesser side B, parallell to the former, and finde it to crosse the line of *Solids* in the points of 400, and such is the proportion betweene the Solids required, which in lesser number is as 5 to 2.

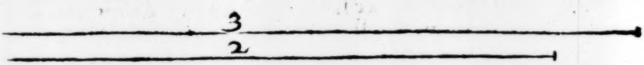
This proposition might haue beene wrought by 60, or any other number that admits seuerall diuisions.

It may also be wrought without opening the *Sector*, for if the sides of the Solids giuen, be applied to the lines of *Solids*, begining allwayes at the center of the *Sector*, there will be such proportion betweene them, as betweene the numbers of parts whereon they fall.

2 *To augment a Solid in a giuen proportion.*
3 *To diminiſh a Solid in a giuen proportion.*

TAke the ſide of the Solid giuen, and to it open the *Sector*, in the points of the number giuen: then keeping the Sector at that angle, the parallell diſtance betweene the points of the number required, ſhall giue the like ſide of the Solid required.

If it be a *parallellopipedon*, or ſome irregular Solid, the other like ſides may be found out in the ſame manner, and with them the Solids required, may be made vp with the ſame angles.



Let *A* be the ſide of a cube, to be augmented in the proportion of 2 to 3. Firſt I take the ſide *A*, and put it ouer in the lines of *Solids* in 2 and 2, ſo the parallell betweene 3 and 3, doth giue me the ſide B, on which if I make a cube, it will haue ſuch proportion to the cube of *A*, as 3 to 2.

In like manner, if B were the diameter of a Sphere, to be diminiſhed in the proportion of 3 to 2. I would take out B, and put it ouer in the lines of *Solids*, in 3 and 3, ſo the parallell betweene 2 and 2, would giue me *A*: to which diameter if I ſhould make a Sphere, it would be leſſe then the Sphere, whoſe diameter is B, in ſuch proportion as 2 is leſſe then 3.

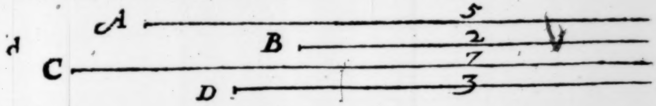
Here alſo for variety of worke, may the like caution be obſerued to that which we gaue in the third *Prop.* of *Lines*.

4 *To adde one like Solid to another.*
5 *To ſubtract one like Solid from another.*

FIrſt the proportion betweene the ſides of the like Solids giuen, is to be found by the firſt *Prop.* of *Solids*: then adde

or

or ſubtract thoſe proportions, and accordingly augment or iminiſh by the former *Prop.*



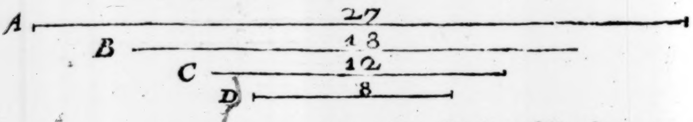
As if *A* and *B* were the ſides of two cubes, and it were required to make a third cube equall to them both : firſt the proportion betweene the ſides *A* and *B*, would be found to be as 100 to 40, or in leſſer termes as 5 to 2. Then becauſe 5 and 2 being added do make 7, I augment the ſide *A* in the proportion of 5 to 7, and produce the ſide *C*, on which if I make a cube, it will be equall to both the cubes of *A* and *B*, which was required.

In like maner *A* and *B* being the ſides of two cubes, if it were required to ſubtract the cube of *B* out of the cube of A, and to make a cube equal to the remainder. Here the proportion being as 5 *to* 2, becauſe 2 taken out of 5, the remainder is 3, I ſhould diminiſh the ſide A in the proportion of 5 to 3, and ſo I ſhould haue the ſide D, on which if I make a cube, it will be equall to the remainder when the cube of B is taken out of the cube of A, that is the two cubes made vpon B and D, ſhall be equall to the firſt cube made vpon the ſide A.

6 *To find two meane proportionall lines betweene two extreme lines giuen.*

FIrſt I find what proportion is betweene the two extreme lines giuen as they are lines, by the fifth *Prop.* of *Lines*, then open the *Sector* in the lines of *Solids*, to the quantitie of the former extreme, and a parallell betweene the points of the number belonging to the other extreme, ſhall be that meane proportionall which is next the former extreme. This done, open the *Sector* againe to this meane proportionall in the points of the former extreme, and the parallell diſtance

betweene the points of the latter extreme, ſhall be the other meane proportionall required.



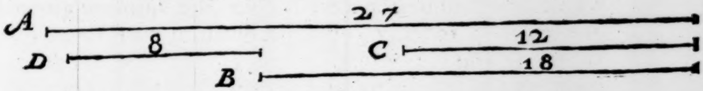
Let the two extreme lines giuen be A and D, the proportion betweene them, as they are lines, will be found to be as 27 to 8. Wherefore I take the line A, and put it ouer in the lines of *Solids* betweene 27 and 27, and keeping the *Sector* at this angle, his parallell betweene 8 and 8, doth giue me B, the meane proportionall next vnto A. Then put I ouer this line B, betweene the aforeſaid 27 and 27, and his parallell betweene 8 and 8 doth giue me the line C, the other meane proportionall which was required.

Againe, for proofe of the operation I put ouer this line C in the aforeſaid 27 and 27, and his parallell between 8 and 8 doth giue me the very line D: whereby it is plaine that theſe foure lines do hold in continuall proportion; and ſo B and C are found to be the meane proportionals betweene A and D the extremes giuen.

7 *To find two meane proportionall numbers between two extreme numbers giuen.*

FIrſt reckon the numbers giuen on both ſides of the lines of *Solids*, beginning from the center, and marking the termes whereto they extend: then take a line out of the line of *Lines*, or any other ſcale of equall parts reſembling the former of thoſe numbers, and put it ouer in the lines of *Solids*, betweene the points of his like number, and a parallell betweene the points belonging to the other extreme, meaſured in the ſcale from whence the other parallell was taken, ſhall giue that meane proportionall number which is next the former extreme. This done, open the *Sector* againe to this meane proportionall in the points of the former extreame, and

and the parallell distance betweene the points of the latter extreame, measured in the same scale as before, shall there shew the other meane proportionall required.



Let the two extreame numbers giuen be 27 and 8; if I shall take the line A, resembling 27 in a scale of eqnall parts, and to it open the *Sector* in 27 and 27, in the line of *Solids*, his parallell betweene 8 and 8 doth giue me B for his next meane proportionall, and this measured in the former scale doth extend to 18. Then put I ouer this line B between the aforesaid 27 and 27, and his parallell between 8 and 8 doth giue me C for the other meane proportionall, and this measured in the former scale doth extend to 12. Againe, for proofe of my worke, I put ouer this line C betweene 27 and 27, as before, and his parallell betweene 8 and 8 doth giue me D, which measured in the former scale doth extend to 8, which was the latter extreame number giuen; whereby it is plaine that these foure numbers do hold in continuall proportion: and therefore 18 and 12 are meane proportionals betweene 27 and 8, which was required.

8 *To finde the cubique roote of a number.*

9 *The roote being giuen to finde the cube number of that roote.*

231. solid Inches in a gallon.

IN the extraction of a cubique root, it is vsuall to set pricks vnder the first figure, the fourth, the seuenth, the tenth, and so forward, omitting two, and pricking the third from the righthand toward the left; and as many pricks as fall to be vnder the cubique number, so many figures shall be in the roote. So that if the number giuen be lesse then 1000, the roote shall be only of one figure; if lesse then 1000000, it shall be but of two figures; if aboue these, and lesse then 1000000000, it shall be but three figures; &c. whereupon

 the

the lines of *Solids* are diuided, first into 1000 parts, and if the numbers giuen be greater thē 1000, the first diuision (whcih before did signifie onely one) must signifie 1000, and the whole line shall be 1000000 : if yet the number giuen be greater then 1000000, the first diuision must now signifie 1000000, and the whole line be esteemed at 1000000000 parts, and if these be to little to expresse the numbers giuen, as oft as wee haue recourse to the begining, the whole line shall encrease it selfe a thousand times.

By these meanes, if the last pricke, to the left hand, shall fall vnder the last figure, the number giuen shall be reckoned at the beginning of the lines of *Solids*, from 1 to 10, and the first figure of the roote shall be alwayes either 1, or 2. If the last pricke shall fall vnder the last figure but one, then the number giuen shall be reckoned in the middle of the line of *Solids*, betweene 10 and 100, and the first figure of the roote shall be alwayes either 2, or 3, or 4. But if the last pricke shall fall vnder the last figure but two, then the number giuen, shall be reckoned at the end of the line of Solids, betweene 100, and 1000.

This being considered when a number is giuen, and the cubique roote required : Set one foote of the compasses in the center of the *Sector*, extend the other in the line of *Solids*, to the points of the number giuen : for this distance applied to one of the line of *Lines*, shall shew what the cubique roote is, without opening the *Sector*.

So the nearest roote of 8490000, is about 204.
The nearest roote of 84900000, is about 439.
The nearest roote of 849000000, is about 947.

On the contrary, a number may be cubed, if first we extend the compasses to the number giuen, in the line of *Lines*, and then apply the distance to the lines of *Solids*; as may appeare by the former examples.

10 *Three numbers being giuen to finde a fourth in a triplicated proportion.*

AS like *Superficies* do hold in a duplicated proportion, so like solids in a triplicated proportion of their homologall sides: and therefore the same worke is to be obserued here on the lines of *Solids*, as before in the lines of *Superficies*; as may appeare by these two examples.

If a cube whose side is 4 inches, shall be 7 pound weight, and it be required to know the weight of a cube whose side is 7 inches; here the proportion would be,

As 4 are to a cube of 7:
so 7 to a cube of $37\frac{1}{2}$.

And if I tooke 7 out of the lines of *Solids*, and put it ouer in 4 and 4, in the lines of *Lines*, his parallell betweene 7 and 7 measured in the lines of *Solids*, would be $37\frac{1}{2}$; and such is the weight required.

If a bullet of 27 pound weight haue a diameter of 6 inches, and it be required to know the diameter of the like bullet, whose weight is 125 pounds; here the proportion would be,

As the cubique root of 27 is vnto 6:
so the cubique root of 125 is vnto 10.

And if I tooke 6 out of the line of *Lines*, and put it ouer in 27 and 27 of the lines of *Solids*, his parallell betweene 125 and 125 measured in the line of *Lines*, would be 10; and such is the length of the diameter required.

The end of the first booke.

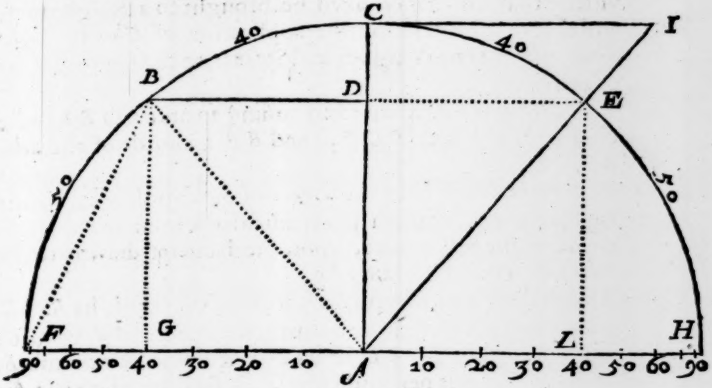
THE SECOND BOOKE OF THE *SECTOR*,

Containing the vse of the Circular *Lines*.

CHAP. I.

Of the nature of Sines, Chords, Tangents *and* Secants, *fit to be knowne before hand in reference to right-line Triangles.*

IN the *Canon* of *Triangles*, a circle is commonly diuided into 360 *degrees*, each *degree* into 60 *minutes*, each *minute* into 60 *seconds*.



A semicircle therefore is an arke of 180 gr.

A quadrant is an arke of 90 *gr.*

The measure of an angle is the arke of a circle, described out of the angular point, intercepted betweene the sides sufficiently produced.

So the measure of a right angle is alwayes an arke of 90 *gr.* and in this example the measure of the angle *B A D* is the arke *B C* of 40 *gr*; the measure of the angle *B A G*, is the arke *B F* of 50 *gr.*

The complement of an arke or of an angle doth commōly signifie that arke which the giuen arke doth want of 90 *gr*: and so the arke *B F* is the cōplement of the arke *B C*; & the angle *B A F*, whose measure is *B F*, is the complement of the angle *B A C*; and on the contrary.

The complement of an arke or angle in regard of a semicircle, is that arke which the giuen arke wanteth to make vp 180 *gr*: and so the angle *E A H* is the complement of the angle *E A F*, as the arke *E H* is the complement of the arke *F E*, in which the arke *C E* is the excesse aboue the quadrant.

The proportions which these arkes (being the measures of angles) haue to the sides of a triangle, cannot be certaine, vnlesse that which is crooked be brought to a straight line; and that may be done by the application of *Chords*, *Right Sines*, *versed Sines*, *Tangents* and *Secants*, to the semidiameter of a circle.

A *Chorde* is a right line subtending an arke: so *B E* is the chorde of the arke *B C E*, and *B F* a chorde of the arke *B F*.

A *right Sine* is halfe the chorde of the double arke, viz. the right line which falleth perpendicularly from the one extreme of the giuen arke, vpon the diameter drawne to the other extreme of the said arke.

So if the giuen arke be *B C*, or the giuen angle be *B A C*, let the diameter be drawne through the center *A* vnto *C*; and a perpendicular *B D* be let downe from the extreme *B*, vpon *A C*; this perpendicular *B D* shall be the *right sine* both of the arke *B C*, and also of the angle *B A C*: and it is also

also the halfe of the chord B E, subtending the arke B C E, which is double to the giuen arke B C. In like maner, the semidiameter F A, is the *right sine* of the arke F C, and of the right angle F A C; for it falleth perpendicularly vpon A C, and it is the halfe of the chord F H.

This whole Sine of *90 gr.* is hereafter called *Radius*; but the other *Sines* take their denomination from the degrees and minutes of their arks.

Sinus versus, the *versed sine* is a segment of the diameter, intercepted betweene the *right sine* of the same arke, and the circumference of the circle. So D C is the *versed sine* of the arke C B, and G F the *versed sine* of the arke B F, and G H the *versed sine* of the arke B H.

A *Tangent* is a right line perpendicular to the diameter, drawne by the one extreme of the giuen arke, and terminated by the *secant* drawne from the center through the other extreme of the said arke.

A *Secant* is a right line drawne from the center, through one extreme of the giuen arke, till it meete with the *tangent* raised from the diameter at the other extreme of the said arke.

So if the giuen arke be C E, or the giuen angle be C A E, let the diameter be drawne through the center A to C, and in C to A C, be raised a perpendicular C I. Then let another line be drawne from the center A through E, till it meet with the perpendicular C I in I; the line C I is a *Tangent*, and A I is the *Secant* both of the arke C E, and of the angle C A E.

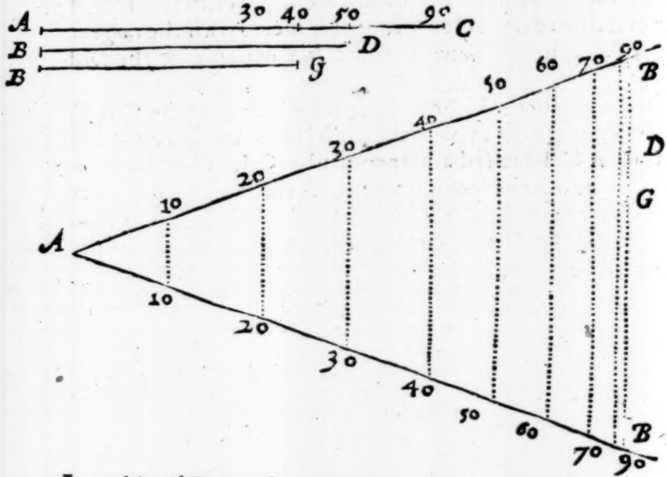
CHAP. II.

Of the generall vse of Sines *and* Tangents.

1 *The Radius being knowne to find the right sine of any arke or angle.*

IF the Radius of the circle giuen be equall to the laterall Radius, that is, to the whole line of *Sines* on the *Sector*, there needs no farther worke, but to take the other sines also out of the side of the *Sector*. But if it be either greater or lesser, then let it be made a parallell Radius, by applying it ouer in the lines of *Sines*, betweene 90 and 90; so the parallell taken from the like laterall sines, shall be the *sine* required.

As if the giuen Radius be *A C*, and it were required to find the sine of 50 *Gr.* & his complement agreeable to that radius,



Let *A B*, *A B* represent the lines of *sines* on the *Sector*, and let B B, the distance betweene 90 and 90, be equall to the giuen

giuen radius *A C*. Here the lines *A* 40, *A* 50, *A* 90, may be called the *laterall sines* of 40, 50, & 90; in regard of their place on the sides of the *Sector*. The lines betweene 40 and 40, betweene 50 and 50, betweene 90 and 90, may be called the *parallell sines* of 40, 50, and 90; in regard they are parallell one to the other. The whole sine of 90 *Gr.* here standing for the semidiameter of the circle, may be called the Radius. And therefore if *A C* be put ouer in the line of *Sines* in 90 and 90, and so made a *parallell radius*, his parallell sine betweene 50 and 50, shall be B D, the sine of 50 required. And because 50 taken out of 90, the complement is 40; his *parallell sine* betweene 40 and 40, shall be B G, the sine of the complement which was required.

2 *The right sine of any arke being giuen to finde the Radius.*

TVrne the sine giuen into a parallell sine, and his parallell *Radius* shall be the *Radius* required.

As if B D were the giuen sine of 50 *Gr.* and it were required to finde the Radius: let B D be made a parallell sine of 50 *Gr.* by applying it ouer in the lines of *Sines*, betweene 50 and 50; so his parallell Radius betweene 90 and 90 shall be A C, the Radius required.

3 *The Radius of a circle, or the right Sine of any arke being giuen, and a streight line resembling a Sine, to find the quantitie of that vnknowne* Sine.

LEt the Radius or right sine giuen be turned into his parallell; then take the right line giuen, and carrie it parallell to the former, till it stay in like *Sines*: so the number of degrees and minutes where it stayeth, shall giue the quantitie of the Sine required.

As if B D were the giuen sine of 50 *Gr.* and B G the streight line giuen: first I make BD a parallell sine of 50 *Gr*; then keeping the *Sector* at this angle, I carie the line B G

parallell, and find it to stay in no other but 40 and 40; and therefore 40 *gr.* is his quantitie required.

4 *The Radius or any right Sine being giuen, to finde the versed sine of any arke.*

IF the arke, whose *versed sine* is required, be lesse then the quadrant, take the sine of the complement out of the radius, and the remainder shall be the *sinus versus*, the versed sine of that arke.

As if A B being the laterall *Radius*, it were required to find to find the versed sine of 40 *gr*; here the sine of the complement is A 50, and therefore B 50 is the *versed sine* required. Or if I reckon from B, at the end of the Sector, toward the center, the distance from 90 to 80, is the versed sine of 10 *gr*; from 90 to 70, the versed sine of 20 *gr*; from 90 to 60, is the versed sine of 30 *gr*; and so in the rest.

If A D be the giuen *sine* of 50 *gr.* and it be required to find the *versed sine* of 50 *gr*; here because A D is vnequall to the laterall sine of 50 *gr*, I make it a parallell. And first I find the radius A C; then the sine of the complement A 40, which being taken out of A C, leaueth C 40 for the versed sine of 50 *gr.* which was required.

But if the arke, whose versed sine is required, be greater then the quadrant, his versed sine also is greater then the *Radius*, by the right sine of his excesse aboue 90 *gr.*

As if A C being the Radius giuen, it were required to find the versed sine of 130 *gr:* here the excesse aboue 90 *gr.* is 40 *gr*; and therefore the versed sine required is equall to the Radius A C and A 40, both being set together.

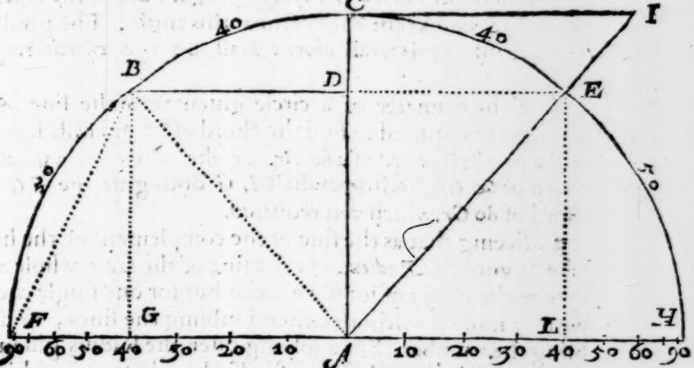
5 *The Diameter or* Radius *being giuen to finde the Chords of euery arke.*

The sines may be fitted many wayes to serue for chords.

1 A *sine* being the halfe of the *chord* of the double arke, if the *sine* be doubled, it giueth the *chord* of the double ark,

a *Sine* of 10 *gr*. doubled giueth a *Chord* of 20 *gr*; and a *Sine* of 15 *gr*. being doubled, giueth a *Chord* of 30 *gr*; and ſo in the reſt. As here B D, the ſine of B C, an arke of 40 *gr*. being doubled giueth B E the chord of B C E, which is an arke of 80 *gr*. Wherefore if the Radius of the circle giuen be equall to the laterall Radius, let the *Sector* be opened neare vnto his length, ſo that both the lines of *Sines* may make but one direct line: ſo the diſtance on the ſines betweene 10 and 10, ſhall be a chord of 20; the diſtance betweene 20 and 20, ſhall be a chord of 40; and the diſtance betweene 30 and 30, ſhall be a chord of 60; and ſo in the reſt.

2 Becauſe a ſine is the halfe of the chord of the double arke, the proportion holdeth.



As the diameter F H vnto the radius A H, ſo the chord B E vnto the ſine D E, or the chord G L vnto the ſine A L: and then if the radius A H, be put for the diameter, which is a chord of 180 gr, the ſine D E or A L ſhall ſerue for a chord of 80 *gr*, and the ſemiradius which is the ſine of 30 *gr*, ſhall ſerue for a chord of 60 *gr*, and go for the ſemidiameter of a circle, and ſo in the reſt. So that by theſe meanes we ſhall not need to double the lines of *Sines* as before, but onely to double the numbers. And to this purpoſe I haue ſubdiuided

each

each degree of the sines into two, that so they might shew how far the halfe degrees do reach in the sines, and yet stand for whole degrees when they are vsed as chords.

Wherefore if the Radius of the circle giuen be equall to the lateral semiradius (the sine of 30 *Gr.* and chord of 60 *Gr.*) there needs no farther work then to take the sine of 10 *Gr.* for a chord of 20 *Gr.* and a sine of 15 *Gr.* for a chord of 30 *Gr.* &c.

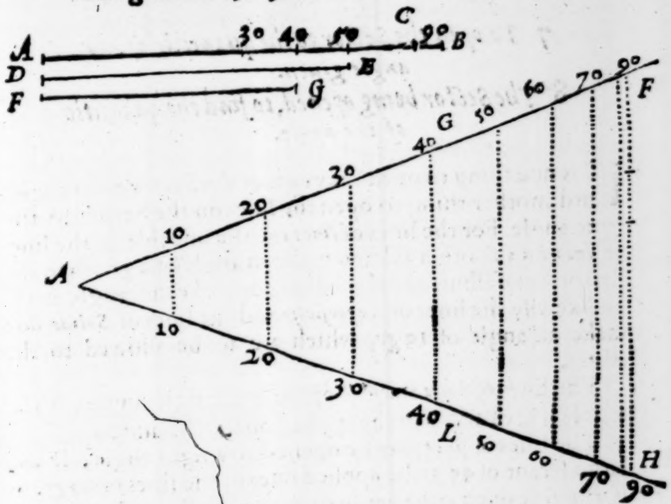
But if the Radius of the circle giuen be either greater or lesser then the laterall semiradius, take the diameter of it, and make it a parallell chord of 180 *Gr.* by applying it ouer the lines of *Sines* between 90 and 90: or take the Radius or Semidiameter which is equall to the chord of 60 *Gr.* and make it a parallell Radius of 60 *Gr.* by applying it ouer in the sines of 30 and 30, and keepe the Sector at this angle. The parallels taken from the laterall *chords* shall be the *chords* required.

As if the diameter of a circle giuen were the line *A B*, and it were required to find the chord of 80 gr: first I make *A B* a parallell chord of 180 *Gr.* or the halfe of it a parallell chord of 60 *Gr*; so his parallell *L G* doth giue me *F G* the chord of 80 *Gr.* which was required.

3 Seeing that as the sine of the complement of the halfe arke is vnto the *Radius*, so the sine of the same whole arke is vnto the chord of it: if we seeke but for one single chord, we may finde it without either doubling the sines, or doubling the number. For applying ouer the Radius giuen in the sine of the complement of halfe the arke required, his parallell sine shall be the chord required.

As if the semidiameter of the circle giuen were *A C*, and it were required to find the chord of 40 *Gr*: the halfe of 40 *Gr.* is 20 *Gr.* the complement of 20 *Gr.* is 70 *Gr.* Wherefore I make *A C* a parallell sine of 70 *Gr.* and his parallell sine *G L* doth giue me *F G* the chord of 40 *Gr.* agreeable to the semidiameter *A C.*

6 *The*



6 *The chord of any arke being giuen to find the diameter and* Radius.

TVrne the chord giuen vnto a parallell chord, and his parallell semiradius shall be the semidiameter, and the parallell radius shall be the diameter.

As if *F G* be the chord of 80 *gr.* I put this ouer in *G* and *L*, the sine of 40, and chord of 80 *gr.* and the parallell chord of 180 *gr.* giueth me *A B* the diameter required.

Or if I turne the chord giuen into a parallell sine of the same quantitie, his parallell sine of the complement of halfe the arke, doth giue me the semidiameter.

As if *F G* be the giuen chord of 40 *gr.* I put it ouer in *G* and *L*, the sines of 40 *gr*; then because the halfe of 40 *gr.* is 20 *gr.* and the complement of 20 *gr.* is 70 *gr.* I take out the parallell sine of 70 *gr.* and it giueth me *A C* for the semidiameter, agreeable to that chord of 40 *gr.*

7 To open the Sector to the quantitie of any angle giuen.

8 The Sector being opened, to find the quantitie of the angle.

IT is one thing to open the edges of the Sector to an angle, and another thing to open the lines on the Sector to the same angle. For the lines of *lines* on the one side, & the lines of *sines* on the other side, do make an angle of 2 *gr.* when the Sector is close shut, and the edges doe make no angle at all. So likewise the lines of *Superficies* and the lines of *Solids* doe make an angle of 10 *gr*, which are to be allowed to the edges.

The lines of *lines* may be opened to a right angle, if the whole line of 100 parts be applied ouer in 80 and 60.

The lines of *sines* may be opened to a right angle, if the large secant of 45 *gr.* be applied ouer in the sines of 90 *gr.* or if the sine of 90 *gr.* be applied ouer in the sines of 45 *gr.* or if the sine of 45 *gr.* be applied ouer in the sines of 30 *gr.*

If it be required to open those lines to any other angle, take out the chord thereof, and apply it ouer in the *semiradius*, and those lines shall be opened to that angle.

As if it were required to open the Sector in the lines of *sines* to an angle of 40 *gr*, take out the chord of 40 *gr*, and to it open the Sector in the chord of 60 *gr*; so shall the lines of *sines* be opened to the angle required. Or if the same chord of 40 *Gr.* be applied ouer betweene 50 and 50, in the lines of *lines*, they shall also be opened to the same angle. If it be applied ouer in 25 of the lines of *Superficies*, or 125 in the lines of *Solids*, they also shall be opened to the same angle: because the chord of 60 *Gr.* or sine of 30 *Gr.* and 50 in the lines of *lines*, and 25 in the lines of *Superficies*, and 125 in the *Solids*, are all of the same length with the semiradius.

Or if the *Semiradius* be applied ouer betweene the sine of 30 *Gr.* and the sine of the complement of the angle required, it will open the lines of *Sines* to that angle.

As

As if the semiradius be applied ouer in the sines of 30 *Gr.* and the sine of 50 *Gr.* it shall open the lines of *Sines* to an angle of 40 *Gr.*

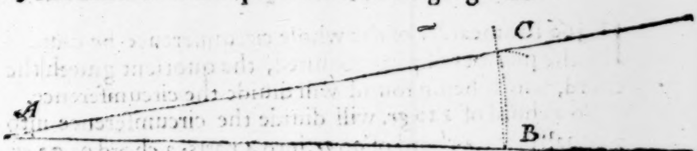
On the contrary, if the *Sector* be opened to an angle, and it be required to know the quantitie thereof, open the compasses to the semiradius, and setting one foote in the sine of 30 *Gr.* turne the other toward the other line of *sines*, and it shall fall there in the complement of the angle; if it fall on 50 *Gr.* the angle is 40 *Gr*; if on 60 *Gr.* the angle is 30 *Gr.* &c.

Or take ouer the parallell chord of 60 *Gr.* and measure it in the laterall chord, and it shall there shew the quantitie of the angle. As if the *Sector* being opened to an angle, I should take ouer the parallell of 30 *Gr.* of the sines, and 60 *Gr.* of the chords, and measure it in the laterall chords, find it to be 40 *Gr*, the angle comprehended betweene the lines of *Sines* is 40 *Gr.* but the angle betweene the edges of the Sector is 2 *Gr.* lesse, and therefore but 38 *Gr.*

9 *To finde the quantitie of any angle giuen.*

IF out of the angular point, to the quantitie of the *Semiradius*, be described an occult arke that may cut both sides of the angle, the chord of this arke measured in the laterall chord, shall giue the quantitie of the angle.

Let the angle giuen be *B A C*: first I take the *Semiradius* with the compasses, and setting one foote in *A*, I cut the sides of the angle in *B* and *C*; then I take the chord *B C*, and measure it in the laterall chord, and I find it to be 11 *Gr.* and 15 *M.* and such is the quantitie of the angle giuen.



Or if the arke be described out of the angular point at any other distance, let the semidiameter be turned into a pa-

rallell chord of 60 *Gr.* then take the chord of this arke, and carrie it parallell till it crosse in like chords: so the place where it stayeth shall giue the quantitie of the angle.

As in the former example, if I make the semidiameter *A B* a parallell chord of 60 *Gr.* and then keeping the *Sector* at that angle, carrie the chord *B C* parallell, till it stay in like chords; I shall finde it to stay in no other but 11 *Gr.* 15 *M.* and such is the angle *B A C.*

10 *Vpon a right line and a point giuen in it, to make an angle equall to any angle giuen.*

First out of the point giuen describe an arke, cutting the same line: then by the 5. *Prop.* afore, find the chord of the angle giuen agreeable to the semidiameter, and inscribe it into this arke: so a right line drawne through the point giuen, and the end of this chord, shall be the side that makes vp the angle.

Let the right line giuen be *A B*, and the point giuen in it be *A*, and let the angle giuen be 11 *gr.* 15 *m.* Here I open the compasses to any semidiameter *A B*, (but as oft as I may conueniently to the laterall semiradius) and setting one foot in *A*, I describe an occult arke *B C*; then I seeke out the chord of 11 *gr.* 15 *m.* and taking it with the compasses, I set one foote in B, the other crosseth the arke in C, by which I draw the line *A C*, and it makes vp the angle required.

11 *To diuide the circumference of a circle into any parts required.*

If 360 the measure of the whole circumference be diuided by the number of parts required, the quotient giueth the chord, which being found will diuide the circumference.

So a chord of 120 *gr.* will diuide the circumference into 3 equall parts; a chord of 90 *gr.* into 4 parts; a chord of 72 *gr.* into 5 parts; a chord of 60 *gr.* into 6 parts; a chord of 51 *gr.* 26. into 7 parts; a chord of 45 *gr.* into 8 parts; a chord of 40 *gr.*

into

into 9 parts; a chord of 36 *gr.* into 10 parts; a chord of 32 *gr.* 44 *m.* into 11 parts; a chord of 30 *gr.* into 12 parts.

In like maner if it be required to diuide the circumference of the circle, whose semidiameter is *A B*, into 32: first I take the semidiameter *A B*, and make it a parallell chord of 60 *gr*; then because 360 *gr.* being diuided by 32, the quotient will be 11 *gr.* 15 *m.* I find the parallell chord of 11 *gr.* 15 *m.* and this will diuide the circumference into 32.

But here the parts being many, it were better to diuide it first into fewer, and after to come ouer it againe. As first to diuide the circumference into 4, and then each 4 parts into 8, or otherwise, as the parts may be diuided.

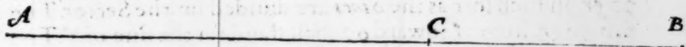
12 *To diuide a right line by extreme and meane proportion.*

THe line to be diuided by extreme and meane proportion, hath the same proportion to his greater segment, as in figures inscribed in the same circle, the side of an *hexagon* a figure of six angles, hath to a side of a *decagon* a figure of ten angles: but the side of a *hexagon* is a chord of 60 *gr.* and the side of a *decagon* is a chord of 36 *gr.*

Let *A B* be the line to be diuided: if I make *A B* a parallell chord of 60 *gr.* and to this semidiameter find *A C* a chord of 36 *gr.* this *A C* shall be the greater segment, diuiding the whole line in *C*, by extreme and meane proportion. So that,

As *A B* the whole line is vnto *A C* the greater segment:
so *A C* the greater segment vnto *C B* the lesser segment.

Or let *A C* be the greater segment giuen: if I make this a parallell chord of 36 *gr.* the correspondent semidiameter shall be the whole line *A C*, and the difference *C B* the lesser segment.



Or let *C B* be the lesser segment giuen: if I make this a parallell chord of 36 *gr.* the correspondent semidiameter

ſhall be greater ſegment *A C,* which added to C B, giueth the whole line A B.

To auoid doubling of lines or numbers, you may put ouer the whole line in the *Sines* of 72 *gr.* and the parallell ſine of 36 *gr.* ſhall be the greater ſegment.

Or if you put ouer the whole line in the ſines of 54 *gr.* the parallell ſine of 30 *gr.* ſhall be the greater ſegment, and the parallell ſine of 18 *gr.* ſhall be the leſſer ſegment.

CHAP. III.

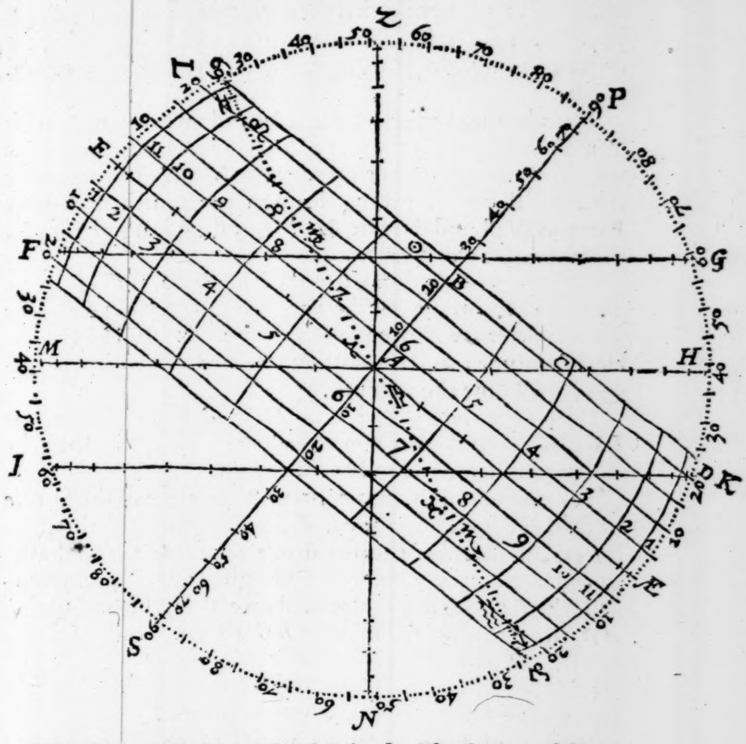
Of the proiection of the Sphere in Plano.

1 THe Sphere may be proiected in *Plano* in ſtreight lines, as in the *Analemma*, if the ſemidiameter of the circle giuen be diuided in ſuch ſort as the line of *Sines* on the Sector.

As if the Radius of the circle giuen were *A E,* the circle thereon deſcribed may repreſent the plane of the generall meridian, which diuided into foure equal parts in *E, P, Æ, S,* and croſſed at right angles with *E Æ* and *P S,* the diameter *E Æ* ſhall repreſent the equator, and *P S* the circle of the houre of 6. And it is alſo the axis of the world, wherein *P* ſtands for the North pole, and *S* for the South pole. Then may each quarter of the meridian be diuided into 90 *gr.* from the equator towards the poles. In which if we number 23 *gr.* 30 *m.* the greateſt declination of the Sun from *E* to ♋ Northwards, from *Æ* to ♑ Southward, the line drawne from ♋ to ♑ ſhal be the ecliptique, and the lines drawne parallell to the equator through ♋ and ♑ ſhall be the tropiques.

Hauing theſe common ſections with the plane of the meridian, if we ſhall diuide each diameter of the Ecliptique into 90 *gr.* in ſuch ſort as the *Sines* are diuided on the Sector. The firſt 30 *gr.* from *A* toward ♋ ſhall ſtand for the ſine of ♈. The 30 *gr.* next following for ♉. The reſt for ♊. ♋. ♌. &c. in their order. So that by theſe meanes we haue the place of the Sun for all times of the yeare.

If



If againe we diuide *A P*, *A S*, in the like ſort, and ſet to the numbers 10. 20. 30. &c. vnto 90 *gr.* the lines drawne through each of theſe degrees parallell to the equator, ſhall ſhew the declination of the Sunne, and repreſent the parallels of latitude.

If farther we diuide *A E*, *A Æ*, and his parallels in the like ſort, and then carefully draw a line through each 15 *gr.* ſo as it makes no angles; the lines ſo drawne ſhall be *ellipticall* and repreſent the houre-circles. The meridian *P E S* the houre

houre of 12 at noone; that next vnto it drawne through 75 *gr*. from the center the houres of 11 and 1, that which is drawne through 60 *gr*. from the center the houres of 10 and 2. &c.

Then hauing respect vnto the latitude, we may number it from *E* Northward vnto *Z*, and there place the zenith: by which and the center the line drawne *Z A N* shall represent the verticall circle, passing through the zenith and nadir East and West, and the line *M A H* crossing it at right angles shall represent the horizon.

These two being diuided in the same sort as the ecliptique and the equator, the line drawne through each degree of the semidiameter *A Z*, parallell to the horizon, shall be the circles of altitude, and the diuisions in the horizon and his parallels shall giue the azimuth.

Lastly, if through 18 *gr*. in *A N*, be drawne a right line *I K* parallell to the horizon, it shall shew the time when the day breaketh, and the end of the twilight.

For example of this proiection, let the place of the Sunne be the last degree of ♉, the parallell passing through this place is *L D*, and therefore the meridian altitude *M L*, and the depression below the horizon at midnight *H D*: the semidiurnall arke *L C*, the seminocturnall arke *C D*, the declination *A B*, the ascentionall difference *B C*, the amplitude of ascenson *A C*. The difference betweene the end of twilight and the day breake is very small; for it seemes the parallell of the Sunne doth hardly crosse the line of twilight.

If the altitude of the Sunne be giuen, let a line be drawne for it parallell to the horizon; so it shall crosse the parallell of the Sunne, and there shew both the azimuth and the houre of the day. As if the place of the Sunne being giuen as before, the altitude in the morning were found to be 20 *gr*. the line *F G* drawne parallell to the horizon through 20 *gr*. in A Z, would crosse the parallell of the Sunne in ☉. Wherefore *F* ☉ sheweth the azimuth, & *L* ☉ the quantitie of houres from the meridian. It seemes to be about halfe an houre past 6 in the morning, and yet more then halfe a point

short

ſhort of the Eaſt.

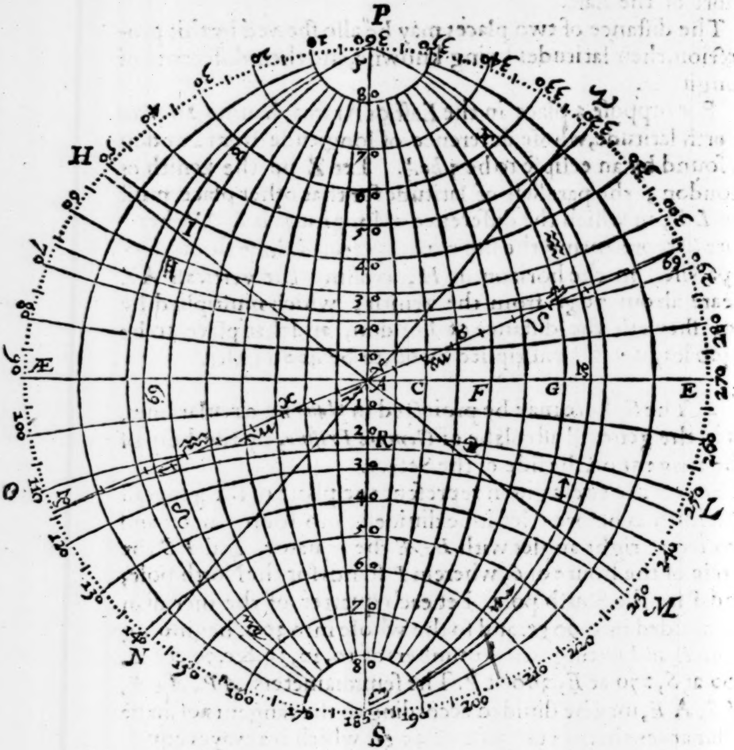
The diſtance of two places may be alſo ſhewed by this proiection, their latitudes being knowne, and their difference of longitude.

For ſuppoſe a place in the Eaſt of Arabia, hauing 20 *gr.* of North latitude, whoſe difference of longitude from London is found by an eclipſe to be 5 *ho.* $\frac{1}{3}$. Let Z be the zenith of London, the parallell of latitude for that other place muſt be *L D*, in which the difference of longitude is *L* ⊙. Wherefore ⊙ repreſenting the ſite of that place, I draw through ⊙ a parallell to the horizon *M H*, croſſing the verticall A Z neare about 70 *gr.* from the zenith, which multiplied by 20, ſheweth the diſtance of London, and that place to be 1400 leagues. Or multiplied by 60, to be 4200 miles.

2 The Sphere may be proiected *in plano* by circular lines, as in the generall aſtrolabe of *Gemma Friſino*, by the help of the tangent on the ſide of the Sector.

For let the circle giuen repreſent the plane of the generall meridian as before; let it be diuided into foure parts, and croſſed at right angles with *E Æ* the equator, and *P S* the circle of the houre of 6, wherein *P* ſtands for the North pole, and *S* for the South pole. Let each quarter of the meridian be diuided into 90 *gr.* and ſo the whole into 360, beginning from *P*, and ſetting to the numbers of 10, 20, 30. &c. 90 at *Æ*, 180 at *S*, 270 at *E*, 360 at *P*. The ſemidiameters *A P*, *A Æ*, *A S*, A *E*, may be diuided according to the tangents of halfe their arkes, that is a tangent of 45 *gr.* which is alwayes equall to the Radius, ſhall giue the ſemidiameter of 90 *gr*; a tangent of 40 *gr.* ſhall giue 80 *gr.* in the ſemidiameter: a tangent of 35 *gr.* ſhall giue 70. &c. So that the ſemidiameters may be diuided in ſuch ſort as the tangent on the ſide of the Sector, the difference being onely in their numbers.

Hauing diuided the circumference and the ſemidiameters, we may eaſily draw the meridians and the parallels by the helpe of the Sector.



The meridians are to be drawne through both the poles *P* and *S*, and the degrees before graduated in the equator. The distance of the center of each meridian from *A* the center of the plane, is equall to the tangent of the same meridian, reckoned from the generall meridian *P Æ S E*, and the semidiameter equall to the secant of the same degree.

As for example, if I should draw the meridian *P B S*, which is the tenth from *P Æ S*, the tangent of 10 *gr.* giueth me *A C*, and the secant of 10 *gr.* giueth me *S C*, whereof *C* is the

center

center of the meridian *P B S*, and *C S* his ſemidiameter: ſo *A F* a tangent of 20 *gr.* ſheweth *F* to be the center of *P D S*, the twentith meridian from *P Æ S*, and *A G* a tangent of 23 *gr.* 30 *M.* ſheweth *G* to be the center of *P* 69 *S.* &c.

The parallels are to be drawne through the degrees, in *A P*, *A S*, and their correſpondent degrees in the generall meridian. The diſtance of the center of each parallell from *A* the center of the plane, is equall to the ſecant of the ſame parallell from the pole, and the ſemidiameter equall to the tangent of the ſame degree. As if I ſhould draw the parallell of 80 *gr.* which is the tenth from the pole *S*, firſt I open the compaſſes vnto *A C* the tangent of 10 *gr.* and this giueth me the ſemidiameter of this parallel, whoſe center is a little from *S*, in ſuch diſtance as the ſecant *S C* is longer then the radius *S A*.

The meridians and parallels being drawne, if we number 23 *gr.* 30 *m.* from *E* to ♋ Northward, from Æ to ♑ Southward, the line drawne from ♋ to ♑ ſhall be the ecliptique: which being diuided in ſuch ſort as the ſemidiameter *A P*, the firſt 30 *gr.* from *A* to ♋ ſhall ſtand for the ſine of ♈; the 30 *gr.* next following for ♉; the reſt for ♊. ♋. ♌. &c. in their order.

If farther we haue reſpect vnto the latitude, we may number it from *E* Northward vnto *Z*, and there place the zenith, by which and the center, the line drawne *Z A N* ſhall repreſent the verticall circle, and the line *M A H* croſſing it at right angles, ſhall repreſent the horizon; and theſe diuided in the ſame ſort as *A P*, the circles drawne through each degree of the ſemidiameter *A Z*; parallell to the horizon, ſhall be the circles of altitude: and the circles drawne through the horizon and his poles, ſhall giue the azimuths.

For example of this proiection, let the place of the Sunne be in the beginning of ♒, the parallell paſſing through this place is ♒ ☉ *L*; and therefore the meridian altitude *M L*, and the depreſſion below the horizon at midnight *H*♒, the ſemidiurnall arke *L* ☉, the ſeminocturnall arke *O* ♒, the declination *AR*, the aſcenſionall difference *R* ☉, the amplitude

tude of ascension *A*☉.

Or if *A* be put to represent the pole of the world, then shall *P Æ S E* stand for the equator, and *P* ♋ *S* ♑ for the ecliptique, and the rest which before stood for meridians, may now serue for particular horizons, according to their seuerall eleuations. Then suppose the place of the Sunne giuen to be 24 *gr.* of ♉, his longitude shall be *PI*, his right ascension *PH*, his declination *HI*. And if the place giuen be 19 *gr.* of ♌, his longitude shal be *PK*, his right ascension *PN*, his declination NK. Againe, the declination brought to the horizon of the place, shall there shew the ascentionall difference, amplitude of ascension, and the like conclusions of the globe. But I intend not here to shew the vse of the Astrolabe, but the vse of the Sector in proiection.

And after this maner may a nocturnall be proiected to shew the houre of the night, whereof I will set downe a type for the vse of Sea-men.

It consists as you see of two parts, the one is a plane, diuided equally according to the 24 houres of the day, and each houre into quarters or minutes, as the plane will beare: the line from the center to XII, stands for the meridian, and XII stands for the houre of 12 at midnight. The other part is a rundle for such starres as are neare the North pole, together with the twelue moneths, and the dayes of each moneth fitted to the right ascension of the starres. Those that haue occasion to see the South pole, may do the like for the Southerne constellations, and put them in a rundle on the back of this plane, and so it may serue for all the world.

The vse of this nocturnall is easie and ready. For looke vp to the pole, and see what starres are neare the meridian, then place the rundle to the like situation, so the day of the moneth will shew the houre of the night.

3 The Sphere may be proiected in *plano* by circular lines, as in the particular Astrolabe of *Ioh. Stophlerin*, by help of the tangent, as before.

For let the circle giuen represent the tropique of ♑, let it be diuided into foure parts, and crossed at right angles with AC the equinoctiall colure, and MB the solstitiall colure, and generall meridian, the center *P* representing the pole of the world. Let each quarter be diuided into 90 *gr.* and so the whole into 360, beginning from *A* towards *B*. The meridian *PM*, or *PB*, may be diuided according to the tangent of halfe his arke. So as the arke from the North pole to the tropique of ♑, being 90 *gr.* and 23 *gr.* 30 *m.* that is 113 *gr.* 30 *m.* and the halfe arke 56 *gr.* 45 *m.* the meridian shall be diuided into 90 *gr.* and 23 *gr.* 30 *m.* in such sort as the tangent of 56 *gr.* 45 *m.* on the side of the Sector is diuided into degrees and halfe degrees; of which *PÆ* the arke of the equator 90 *gr.* from the pole, shall be giuē by the tangent of 45 *gr.* And *P* 69 the arke of the Summer tropique 66 *gr.* 30 *m.* from the pole, shall be giuen by the tangent of 33 *gr.* 15 *m.* And the circles drawne vpon the center *P* through *Æ* and ♋, shall be the equator, and the Summer tropique.

Hauing the equator and both the tropiques, the eclip-

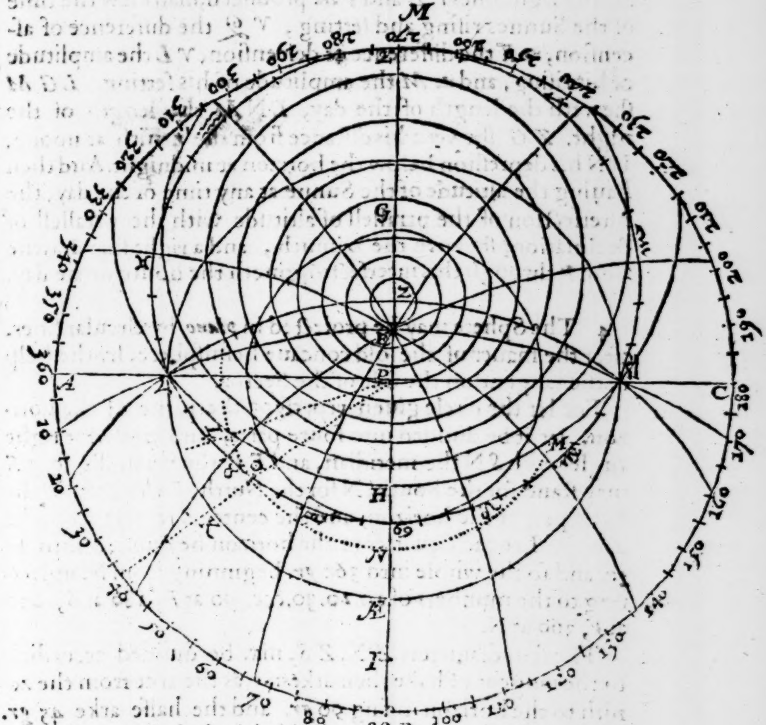
tique ♈ ♋ ♎ ♑ shall be drawne from the one tropique to the other, through the intersection of the equator and the equinoctiall colure. And it may be diuided first into the twelue Signes after this maner: *P E* the arke of the pole of the ecliptique 23 *gr.* 30 *m.* from the pole of the world, shall be giuen by the tangent of 11 *gr.* 45 *m.* The center of the circle of longitude passing through this pole *E* ♈ and ♎, shal be found at *D* (somewhat below *B*) by the tangent of 66 *gr.* 30 *m.* Then through *D* draw an occult line parallell to *A C*, and diuide it on each side from *D*, in such sort as the tangent is diuided on the side of the *Sector*, allowing 45 *gr.* to be equall to *D E*. So the thirtith degree from D toward the right hand, shall be the center of the circle of longitude passing through *E* ♉ and ♏. The sixtith degree, the center of ♊ *E* ♐. The thirtith degree from D toward the left hand, the center of ♓ *E* ♍. The sixtith, the center of ♒ *E* ♌. And the other intermediate degrees shal be the centers to diuide each Signe into 30 *gr.*

If farther we haue respect vnto the latitude, we may (the meridian being before diuided) number it from *P* Northward vnto *H*, and there place the North intersection of the meridian and horizon: then the complement of the latitude being numbred from *P* Southward vnto *Z*, shall there giue the zenith; and 90 *gr.* from *Z* Southward vnto *F*, shall there giue the South intersection of the meridian and horizon. The middle betweene *F* and *H* shall be *G* the center of the horizon ♈ *H* ♎ *F*, passing through the beginning of ♈ and ♎, vnlesse there be some former error.

All parallels to the horizon may be found in like sort by their intersections with the meridian, and the middle betweene those intersections is alwayes the center.

The azimuths may be drawne as the circles of longitude were before. For the center of the first verticall ♈ *Z* ♎ will be found at *I* (somewhat neare vnto *B*) by the tangent of the latitude. And if through *I* we draw an occult line parallell to *A C*, and diuide it on each side from *I*, in such sort as the tangent is diuided on the side of the *Sector*, allowing 45 *gr.* to be

equall



equall to *)Z*; these diuisions shall be the centers, and the distance from these diuisions vnto *Z*, shall be the semidiameters whereon to describe the rest of the azimuths.

For example of this proiection, let ☉ the place of the Sun giuen be 10 *gr.* of ♉: a right line drawne from *P* through this place vnto the equator, shall there shew his right ascension ♈*K*, and his declination *K* ☉. Then may we on the center *P* and semidiameter ☉*P*, draw an occult parallell of declinatiō, crossing the horizon in *L* and *M*, the meridian in *G* and N.

So

So the right lines *P L* and *P M* produced, shall shew the time of the Sunnes rising and setting, ♈ *Q* the difference of ascension, ♎ *R* the difference of descension, ♈ *L* the amplitude of his rising, and ♎ *M* the amplitude of his setting. *L G M* sheweth the length of the day, *L* N *M* the length of the night. *Z G* sheweth his distance from the zenith at noone, H N his depression below the horizon at midnight. And then hauing the altitude of the Sunne at any time of the day, the intersection of the parallell of altitude with the parallell of declination, sheweth the azimuth, and a right line drawne from *P* through this intersectiõ, giueth the houre of the day.

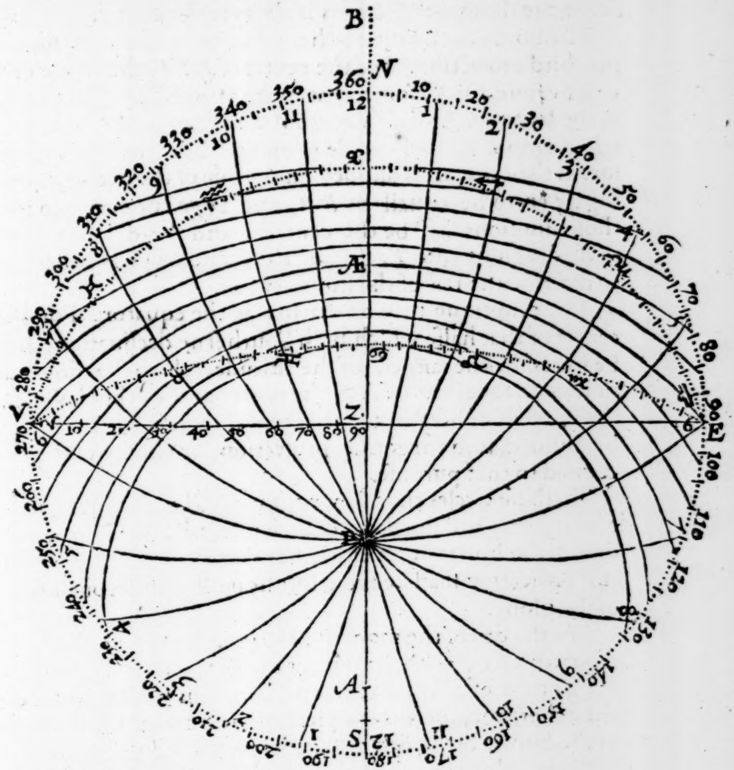
4 The Sphere may be proiected in *plano* by circular lines, after the maner of the old concaue hemisphere, by the help of the tangent on the side of the Sector.

For let the circle giuen represent the plane of the horizon, let it be diuided into foure parts, and crossed at right angles with *S* N the meridian, and *E V* the verticall; so as *S* may stand for the South, N for the North, *E* the East, *V* the West part of the horizon, and the center *Z* representeth the zenith. Let each quarter of the horizon be diuided into 90 *gr.* and so the whole into 360 *gr.* beginning from N, and setting to the numbers of 10. 20. 30. &c. 90 at *E*, 180 at *S*, 270 at *V*, 360 at N.

The semidiameters *Z* N, *Z S*, may be diuided according to the tangent of halfe their arkes: So as the arke from the zenith to the horizon being 90 *gr.* and the halfe arke 45 *gr.* the semidiameters are to be diuided in such sort as the tangent of 45 *gr.* as was shewed before in the second proiection. And if from *Z* we draw circles through each of these diuisions, they shall be parallels of altitude.

Then hauing respect vnto the latitude, we may (the meridian being before diuided) number it from *Z* to *Æ*, and there place the intersection of the meridian and equator. The complement of the latitude from *Z* vnto *P*, shall there giue the pole of the world, and 90 further from *P* shall there giue the other intersection of the meridian and equator.

The



The middle betweene these intersections shall be *A*, the center of the equator, passing through *E* and *V*, vnlesse there be some former error. The intersections of the tropiques depend on the equator. From *Æ* 23 *gr*. 30 *m*. farther shal be ♑, the intersection of the meridian & the Southerne tropique. From *Æ* 23 *gr*. 30 *m*. nearer shall be ♋, the intersection of the meridian and the Northerne tropique. The intersections of the other intermediat parallels, shall be giuen in like sort, by their degrees of distance from the equator, and the middle

betweene those intersections is alwayes the center.

The houre circles may be here drawne as the azimuths in the third proiection. For the center of *E P V*, the houre of 6 will be found at *B* (somewhat neare vnto N) by the tangent of the latitude. And if through *B* we draw an occult line parallell vnto *E V*, and diuide it on each side from *B*, in such sort as the tangent is diuided on the side of the Sector, allowing 45 *gr.* to be equall to *B P*, and 15 *gr.* for euery houre: those diuisions shall be the centers, and the distance from these diuisions vnto *P*, shall be the semidiameters, whereon to describe the rest of the houre circles.

The ecliptique may be drawne as the equator. For the center of that halfe which hath Southerne declination, shall be giuen by the tangent of the altitude, which the Sun hath in his entrance into ♑. And the center of the other halfe, by the tangent of his altitude, at his entrance into ♋. And it may be diuided, as in the former proiection, or else by tables calculated to that purpose.

To these circles thus drawne, if we shall adde the moneths of the yeare, and the dayes of each moneth, as we may well doe, at the horizon, on either side betweene the tropiques; this proiection shall be fitted for the most vsefull conclusions of the globe.

For the day of the moneth being giuen, the parallell that shooteth on it, doth shew what declination the Sunne hath at that time of the yeare. And where this parallell crosseth the ecliptique, there is the place of the Sunne. Or the place of the Sunne being first giuen, the parallell which crosseth it shall at the horizon shew the day of the moneth. Either of these then being giuen, or onely the parallell of declination, we may follow it first vnto the horizon, there the distance of the end of the parallell from *E* or *V*, sheweth the amplitude; the same among the houre circles sheweth the time when the Sunne riseth or setteth. Then hauing the altitude of the Sunne at any time of the day, the intersection of the parallell of declination with the parallell of altitude, sheweth the houre of the day; and a right line drawne from Z

through

through this intersection to the horizon, giueth the azimuth.

Thus in either of these proiections, that which is otherwise most troublesome, is easily done by the help of the *tangent* line: and what I haue said of this line, the same may be wrought by scale and numbers out of the table of Tangents.

CHAP. IV.

Of the resolution of right-line Triangles.

IN all Triangles there being six parts, viz. three angles and three sides, any three of them being giuen, the rest may be found by the Sector.

As in a Rectangle triangle,
1 *To finde the base, both sides being giuen.*

Let the *Sector* be opened in the lines of *Lines* to a right angle, (as before was shewed *Cap.2. Prop.7.*) then take out the sides of the triangle, and lay them, one on one line, the other on the other line, so as they meete in the center, and marke how farre they extend. For the line taken from the termes of their extension, shall be the base required, viz. the side opposite to the right angle.

Or adde the squares of the two sides (as in *Prop.*4. Superf.) and the side of the compound square shall be the base.

2 *To find the base by hauing the angles, and one of the sides giuen.*

Take the side giuen, and turne it into the parallell sine of his opposite angle; so the parallell Radius shall be the base.

3 *To find a side by hauing the base, and the other side giuen.*

Let the Sector be opened in the lines of *lines* to a right angle,

angle, and the side ginen laid on one of those lines from the center; then take the base with a paire of compasses, and setting one foote in the terme of the giuen side, turne the other to the other line of the Sector, and it shall there shew the side required.

Or take the square of the side out of the square of the base (as in *Prop.4. Superf.*) and the side of the remaining square shall be the side required.

4 *To find a side hauing the base and the angles giuen.*

Take the base giuen, and make it a parallell Radius, so the parallell *sines* of the angles, shall be the opposite sides required.

5 *To find a side by hauing the other side and the angles giuen.*

Take the side giuen, and turne it into his parallell *sine* of his opposite angle; so the parallell *sine* of the complement shall be the side required.

6 *To find the angles by hauing the base and one of the sides giuen.*

First take out the base giuen, and laying it on both sides of the Sector, so as they may meete in the center, and marke how farre it extendeth. Then take out the laterall Radius, and to it open the Sector in the termes of the base. This done, take out the side giuen, and place it also on the same lines of the Sector from the center. For the parallell taken in the termes of this side, shall be the sine of his opposite angle.

Or take the base giuen, and make it a parallell Radius; then take the side giuen, and carrie it parallell to the base, till it stay in like *sines*: so they shall giue the quantitie of the

the opposite angle.

7 *To finde the angles by hauing both the sides giuen.*

Take out the greater side, and lay it on both sides of the Sector, so as they meete in the center, and marke how farre it extendeth. Then take the other side, and to it open the Sector in the termes of the greater side; so the parallell Radius shall be the tangent of the lesser angle. The third angle is alwayes knowne by the complement.

8 *The Radius being giuen, to find the tangent, and secant of any arke.*
9 *The tangent of any arke being giuen, to find the tangent thereof, and the Radius.*
10 *The secant of any arke being giuen, to find the tangent thereof, and the Radius.*

The tangent, and the secant, together with the Radius of euery arke, do make a right angle triangle; whose sides are the *Radius* and *tangent*, and the base alwayes the *secant*; and the angles alwayes knowne by reason of the giuen arkes. Wherefore the solution is the same with those before.

In any right-lined triangle whatsoeuer,
11 *To find a side by knowing the other two sides, and the angle contained by them.*

Let the *Sector* be opened in the lines of *lines* to the angle giuen, then take out the sides of the triangle, & laying them the one on the one line, the other on the other, so as they meete in the center, marke how far they extend. For the line taken betweene the termes of their extension, shall be the third side required.

12 *To find a side by hauing the other two sides, and one of the adiacent angles, so it be knowne which of the other angles is acute or oblique.*

Let the *Sector* be opened in the lines of *lines* to the angle giuen, and the adiacent side layd on one of those lines from the center; then take the other side with a paire of compasses, and setting one foote in the terme of the former giuen side, turne the other to the other line of the *Sector* which here representeth the side required, and it shall crosse it in two places; but with which of them is the terme of the side required, must be iudged by the angle.

As if in the triangle following, the side *A C* being giuen, and the side *C D* and the angle *C A D* 18 *gr.* 40 *m.* it were required to find the side *A D.*

First I open the Sector in the lines of *lines* to an angle of 18 *gr.* 40 *m.* and laying the adiacent side from the center *A*, it extendeth to 800 in *C*. Then I take the other side *CD* with the compasses, and setting one foote in *C*, and turning the other to the other line of the *Sector*, I find that it doth crosse it both in *B* and D; so that it is vncertaine whither the side required be *AB* or *A* D, onely it may be iudged by the angle. For if the inward angle where they crosse be obtuse, the side required is the lesser; if it be acute, it is the greater.

13 *To find a side by hauing the angles and one of the other sides giuen.*

Take the side giuen, and turne it into the parallell sine of his opposite angle; so the parallell sines of the other angle shall be the opposite sides required.

14 *To find the proportion of the sides by hauing the three angles.*

Take the laterall sines of the angles, and measure them in the line of *lines*. For the numbers belonging to those lines do giue the proportion of the sides.

15 *To finde an angle by knowing the three sides.*

Let the two containing sides be layd on the lines of the *Sector* from the center, one on one line, and the other on the other; and let the third side, which is opposite to the angle required, be fitted ouer in their termes: so shall the *Sector* be opened in those lines to the quantitie of the angle required. The quantitie of this angle is found as in *Cap.2. Prop.8.*

16 *To finde an angle by hauing two sides and one adiacent angle.*

First take out the side opposite to the angle giuen, and laying it on both sides of the Sector, so as they meete in the center, marke how farre it extendeth; then take out the laterall sine of the angle, and to it open the Sector in the termes of the first side: this done, take out the other side giuen, and place it also on the same lines of the Sector from the center, for the parallels taken in the termes of this side, shall be the sine of the angle opposite to the second side.

Or take out the side opposite to the angle giuen, and make it a parallell sine of that angle; then take the other side giuen and carrie it parallell to the former, till it stay in like sines: so they shall giue the quantitie of the angle opposite to the second side.

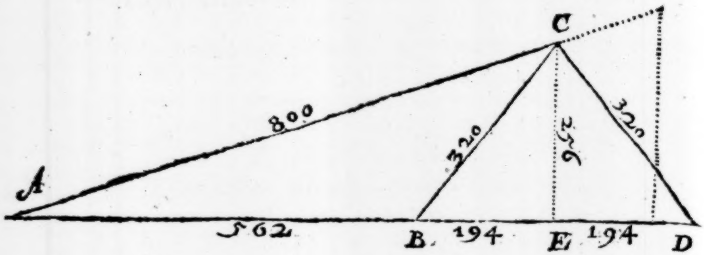
17 *To*

17 *To finde an angle by hauing two sides, and the angle cont ained by them.*

First find the third side by the 11.*Prop.* and then the angles may be found by the 15.or 16.*Prop.*

For practise in each of these cases, we may vse the examples following, wherein *C E A, C E B, C E D* are rectangle in *E*; the rest consist of oblique angles.

C A B	18*gr.*	40*m.*
A B C	126	52
A C B	34	28
A C D	108	12
A D C	53	8
B C D	73	44



For obseruation of angles, the *Sector* may haue sights set on the moueable foote; so that by looking through them, the edges of the Sector may be applied to the sides of the angle.

For

For measuring of the ſides of leſſer triangles, any ſcale may ſuffice, either of feete, or inches, or leſſer parts. But for greater triangles, eſpecially for plotting of grounds, I hold it fit to vſe a chaine of foure perches in length, diuided into an hundred links. For ſo the length being multiplied into the bredth, the fiue laſt figures giue the content in roods and perches by this Table; the other figures toward the left hand, doe ſhew the number of acres directly.

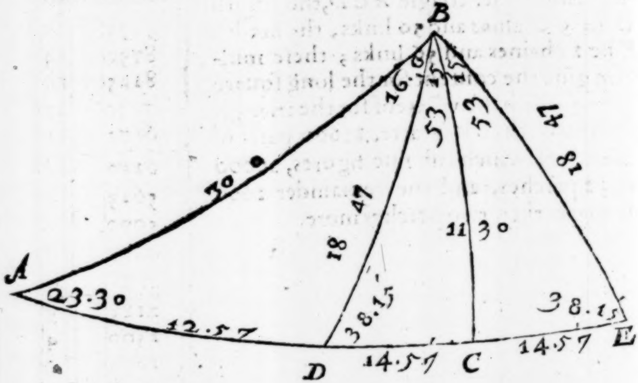
As if in the former triãgle *ACD*, the length *A* D be 9 chaines and 50 links, the bredth *CE* be 2 chaines and 56 links; theſe multiplied giue the content for the long ſquare 2.43200, the halfe whereof for the triangle is 1.21600, that is 1 acre, 21600 parts of 100000, of which laſt fiue figures, 20000 giue 32 perches, and the remainder 1600 giue better then two perches more.

Links	*R*	*P*
100000	4	0
90000	3	24
80000	3	8
70000	2	32
60000	2	16
50000	2	0
40000	1	24
30000	1	8
20000		32
10000		16
9375		15
8750		14
8125		13
7500		12
6875		11
6250		10
5625		9
5000		8
4375		7
3750		6
3125		5
2500		4
1875		3
1250		2
625		1

CHAP. V.

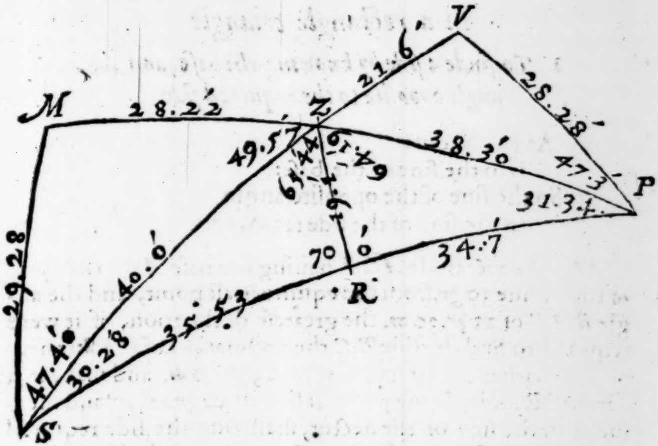
Of the resolution of sphericall triangles.

FOr our practise in sphericall triangles, let *A* be the equinoctiall point, *A* B an arke of the ecliptique representing the longitude of the Sunne in the beginning of ♉, *B C* an arke of the declination from the Sunne to the equator, and *A C* an arke of the equator representing the right ascension.



Let B D be an arke of the horizon representing the amplitude of the Sunnes rising from the East, and *B E* an arke of the horizon for his setting from the West: so D C shall be the difference of ascension, and *C E* the difference of descension; A D the oblique ascension, and *A E* the oblique descension of the same place of the Sunne in our latitude at *Oxford* of 51 *gr.* 45 *m.* whose complement 38 *gr.* 15 *m.* is the angle at *E* and D. The triangles *A C B*, D C *B*, *E C B*, are rectangle in *C*: the other *A* D *B*, *A E B*, consist euery way of oblique angles.

Or



Or to fit an example nearer to the latitude of *London*. Let *Z P S* represent the zenith pole and Sun, *Z P* being 38 *gr.* 30 *m.* the complement of the latitude, *P S* 70 *gr.* the complement of the declination, and *Z S* 40 *gr.* the complement of the Suns altitude. The angle at *Z* shall shew the azimuth, and the angle at *P*, the houre of the day from the meridian. Then if from *Z* to *P S* we let downe a perpendicular *Z R*, we shall reduce the oblique triangle into two rectangle triangles *Z R P*, *Z R S*. Or if from *S* to *Z P* we set downe a perpendicular *S M*, we shall reduce the same *Z P S* into two other triangles, *S M Z*, *S M P*, rectangle at *M*: whatsoeuer is said of any of these triangles, the same holdeth for all other triangles in the like cases.

For the resolution of each of these, there be seueral wayes. I onely chuse those which are fittest for the *Sector*, wherein if that be remembred which before is shewed in the generall vse of the *Sector* concerning laterall and parallell entrance, it may suffice onely to set downe the proportion of the three parts giuen to the fourth required, and so I shew first by the *sines* alone.

In a rectangle triangle

1 *To finde a side by knowing the base, and the angle opposite to the required side.*

As the Radius
is to the sine of the base:
So the sine of the opposite angle
to the sine of the side required.

As in the rectangle *A C B*, hauing the base *AB*, the place of the Sunne 30 *gr.* from the equinoctiall point, and the angle *B A C* of 23 *gr.* 30 *m.* the greatest declination, if it were required to find the side *B C* the declination of the Sunne.

Take either the laterall sine of 23 *gr.* 30 *m.* and make it a parallell Radius; so the parallell sine of 30 *gr.* taken and measured in the side of the Sector, shall giue the side required 11 *gr.* 30 *m.* Or take the sine of 30 *gr.* and make it a parallell Radius; so the parallell sine of 23 *gr.* 30 *m.* taken and measured in the laterall sines, shall be 11 *gr.* 30 *m.* as before.

So in the triangle Z *P S* hauing Z *P* 38 *gr.* 30 *m.* and the angle *P* 31 *gr.* 34 *m.* giuen, we shall find the perpendicular Z *R* to be 19 *gr.* 1 *m*; or hauing *P S* 70 *gr.* and the said angle *P* 31 *gr.* 34 *m.* giuen, we may finde the perpendicular *S M* to be 29 *gr.* 28 *m.*

2. *To finde a side by knowing the base and the other side.*

As the sine of the complement of the side giuen
is to the Radius:
So the sine of the complement of the base
to the sine of the complement of the side required.

So in the rectangle *A C B*, hauing *A B* 30 *gr.* and *BC* 11 *gr.* 30 *m.* giuen, the side *A C* will be found 27 *gr.* 54 *m.*

Or in the rectangle *Z R P*, hauing Z *P* 38 *gr.* 30 *m.* and Z *R* 19 *gr.* 1 *m.* giuen, the side *R P* will be found 34 *gr.* 7 *m.*

3 To

3 *To find a side by knowing the two oblique angles.*

As the sine of either angle
to the sine of the complement of the other angle:
So is the Radius
to the sine of the complement of the side opposite
to the second angle.

So in the rectangle *A C B*, hauing *C A B* for the first angle 23 *gr.* 30 *m.* and *A B C* for the second 69 *gr.* 21 *m.* the side *A C* will be found 27 *gr.* 54 *m.* Or making *A B C* the first angle, and *C A* B the second, the side BC will be found 11 *gr.* 30 *m.*

4. *To finde the base by knowing both the sides.*

As the Radius
to the sine of the complement of the one side:
So the sine of the complement of the other side,
to the sine of the complement of the base required.

So in the rectangle *A C*B hauing *A C* 27 *gr.* 54 *m.* and BC 11 *gr.* 30 *m.* the base *A* B will be found 30 *gr.*

5 *To finde the base by knowing the one side, and the angle opposite to that side.*

As the sine of the angle giuen,
to the sine of the side giuen:
So is the Radius
to the sine of the base required.

So in the rectangle B C D, knowing the latitude and the declination, we may find the amplitude; as hauing B C the side of the declination 11 *gr.* 30 *m.* and B D C the angle of the complement of the latitude 38 *gr.* 15 *m.* the base B D which is the amplitude, will be found to be 18 *gr.* 47 *m.*

6 *To find an angle by the other oblique angle, and the side opposite to the inquired angle.*

As the Radius
to the sine of the complement of the side:
So the sine of the angle giuen,
to the sine of the complement of the angle required.

So in the rectangle A C B, hauing the angle B A C 23 *gr.* 30 *m.* and the side A C 27 *gr.* 54 *m.* the angle A B C will be found 69 *gr.* 21 *m.*

7 *To finde an angle by the other oblique angle, and the side opposite to the angle giuen.*

As the sine of the complement of the side
to the side of the complement of the angle giuen:
So is the Radius
to the sine of the angle required.

So in the rectangle A C B, hauing B A C 23 *gr.* 30 *m.* and B C 11 *gr.* 30 *m.* the angle A B C will be found 69 *gr.* 21 *m.*

8 *To finde an angle by the base, and the side opposite to the inquired angle.*

As the sine of the base
is to the Radius:
So the sine of the side
to the sine of the angle required.

So in the rectangle B C D, hauing B D 18 *gr.* 47 *m.* and B C 11 *gr.* 30 *m.* the angle B D C will be found 38 *gr.* 15 *m.*

These eight Propositions haue been wrought by the *sines* alone; those which follow require ioynt help of the *tangent.*

And forasmuch as the *tangent* could not well be extended beyond 63 *gr.* 30 *m.* I shall set downe two wayes for the resolution of each Proposition; if the one will not hold, the other may.

9 *To*

9 *To finde a side by hauing the other side, and the angle opposite to the inquired side.*

1 As the Radius
to the sine of the side giuen:
So the tangent of the angle,
to the tangent of the side required.

2 As the sine of the side giuen,
is to the Radius:
So the tangent of the complement of the angle,
to the tangent of the complement of the side required.

So in the rectangle ACB, hauing the right side AC 27 *gr.* 54 *m*, and the angle BAC 23 *gr.* 30 *m.* the side BC will be found to be 11 *gr.* 30 *m.*

10 *To find a side, by hauing the other side, and the angle adiacent next to the inquired side.*

1 As the tangent of the angle,
to the tangent of the side giuen:
So is the Radius
to the sine of the side required.

2 As the tangent of the complement of the side,
to the tangent of the complement of the angle:
So is the Radius
to the sine of the side required.

This and the like, where the tangent standeth in the first place, are best wrought by parallell entrance. And so in the rectangle BCD, hauing BC the side of declination 11 *gr.* 30 *m.* and BDC the angle of the complement of the latitude 38 *gr.* 15 *m.* the side DC, which is the ascensionall difference, will be found 14 *gr.* 57 *m.*

By the ascensionall difference is giuen the time of the Sunnes rising and setting, and length of the day; allowing an

an houre for each 15 *gr.* and 4 minutes of time for each seuerall degree. As in the example the difference betweene the Sunnes ascension in a right sphere, which is alwayes at 6 of the clocke, and his ascension in our latitude being 14 *gr.* 57 *m.* it sheweth that the Sunne riseth very neare an houre before 6, because of the Northerne declination; or after 6, if the Sunne be declining to the Southward.

11 *To finde a side by knowing the base, and the angle adiacent next to the inquired side.*

1 As the Radius
to the sine of the complement of the angle:
So is the tangent of the base,
to the tangent of the side required.

2 As the sine of the complement of the angle
is to the Radius:
So the tangent of the complement of the base,
to the tangent of the complement of the side required.

So in the rectangle A C B, knowing the place of the Sun from the next equinoctiall point, and the angle of his greatest declination, we may find his right ascension: viz. the base A B 30 *gr.* and the angle BAC 23 *gr.* 30 *m.* being giuen, the right ascension A C will be found 27 *gr.* 54 *m.*

12 *To finde the base by knowing the oblique angles.*

As the tangent of the one angle,
to the tangent of the complement of the other angle:
So is the Radius
to the sine of the complement of the base.

So in the rectangle A C B, hauing B A C 23 *gr.* 30 *m.* and A B C 69 *gr.* 21 *m.* the base A B will be found 30 *gr.*

13 *To*

13 *To finde the base, by one of the sides, and the angle adiacent next that side.*

1 As the Radius
is to the sine of the complement of the angle:
So the tangent of the complement of the side,
to the tangent of the complement of the base.

2 As the sine of the complement of the angle
is to the Radius:
So the tangent of the side giuen,
to the tangent of the base required.

So in the rectangle *A C B*, hauing *A C 27 gr.54 m.* and *B A C 23 gr.30 m.* the base A B will be found 30 *gr.0 m.*

14 *To find an angle, by knowing both the sides.*

1 As the Radius
is to the sine of the side next the inquired angle:
So the tangent of the complement of the opposite side,
to the tangent of the complement of the angle required.

2 As the sine of the side next the inquired angle,
is to the Radius:
So the tangent of the opposite side,
to the tangent of the angle required.

So in the rectangle A C B, hauing A C 27 *gr.54 m.* and B C 11 *gr.30 m.* the angle at *A* will be found 23 *gr.30 m.* and the angle at B 69 *gr.21 m.*

15 *To finde an angle, by the base, and the side adiacent to the inquired angle.*

1 As the tangent of the complement of the side,
to the tangent of the complement of the base:

So is the Radius
to the sine of the complement of the angle required.

2 As the tangent of the base,
to the tangent of the side:
So is the Radius,
to the sine of the complement of the angle required.

So in the rectangle BCD, hauing the base BD 18 *gr.* 47 *m.* and the side BC 11 *gr.* 30 *m.* the angle D B C between them will be found 53 *gr.* 15 *m.*

16 *To find an angle, by knowing the other oblique angle, and the base.*

1 As the Radius,
to the sine of the complement of the base:
So the tangent of the angle giuen,
to the tangent of the complement of the angle required.

2 As the sine of the complement of the base,
is to the Radius:
So the tangent of the complement of the angle giuen,
to the tangent of the angle required.

So in the rectangle A C B, hauing the angle at *A* 23 *gr.* 30 *m.* and the base A B 30 *gr.* the angle A B C will be found 69 *gr.* 21 *m.*

These sixteen cases are all that can fall out in a rectangle triangle: those which follow do hold

In any sphericall triangle whatsoeuer

17 *To find a side opposite to an angle giuen, by knowing one side, and two angles, whereof one is opposite to the giuen side, the other to the side required.*

As the sine of the angle opposite to the side giuen,
is to the sine of that side giuen:
So the sine of the angle opposite to the side required,
to the sine of the side required.

So in the triangle A B E, hauing the place of the Sunne, the latitude, and the greatest declination, we may finde the amplitude. As hauing A B 30 *gr.* BA E 23 *gr.* 30 *m.* and AEB 38 *gr.* 15 *m.* the side B E which is the amplitude, will be found 18 *gr.* 47 *m.*

18 *To finde an angle opposite to a side giuen, by hauing one angle and two sides, the one opposite to the giuen angle, the other to the angle required.*

As the sine of the side opposite to the angle giuen,
is to the sine of that angle giuen:
So the sine of the side opposite to the angle required,
to the sine of the angle required.

So in the triangle *Z P S*, hauing the azimuth, and latitude, and declination, we may find the houre of the day. As hauing *P Z S* 130 *gr.* 3 *m.* *PS* 70 *gr.* and *Z S* 40 *gr.* the angle *Z P S*, which sheweth the houre from the meridian shall be found 31 *gr.* 34 *m.*

19 *To find an angle by knowing the three sides.*

This proposition is most vsefull, but most difficult of all others: as in Arithmetique, so by the ~~*Sector, yet*~~ may it be performed seuerall wayes.

1 According to *Regiomontanus* and others.

As the sine of the lesser side next the angle required,
to the difference of the versed sines of the base and difference of the sides:
So is the Radius
to a fourth proportionall.

 Then

Then as the sine of the greater side next the angle required
is to that fourth proportionall :
So is the Radius
to the versed sine of the angle required.

So in the triangle *ZPS*, hauing the side *P S*, the cōplement of the declination 70 *gr.* 0 *m.* the side *Z P* the complement of the latitude 38 *gr.* 30 *m.* and the base *Z S* the complement of the altitude 40 *gr.* the angle of the houre of the day *ZPS* will be found 31 *gr.* 34 *m.* which is 2 *h.* 6 *m.* from the meridian.

For the base being 40 *gr.* 0 *m.* and the difference of the sides 38 *gr.* 30 *m.* and 70 *gr.* 0 *m.* being 31 *gr.* 30 *m.* the difference of their versed sines wil be the same with the distance between the right sine of 50 *gr.* and 58 *gr.* 30 *m.* This difference I take out, and make it a parallell sine of the lesser side 38 *gr.* 30 *m.* so the parallell Radius wil be the fourth proportionall. Then coming to the second operation, I make this fourth proportionall, a parallel sine of the greater side of 70 *gr.* 0 *m.* and take out his parallell Radius. For this measured from 90 *gr.* toward the center, will be the versed sine of 31 *gr.* 34 *m.*

In the like sort in the same triangle *Z P S*, hauing the same complements giuen, the angle *P Z S* which is the azimuth from the North part of the meridian, will be found 130 *gr.* 3 *m.* For here the base opposite to the angle required being 70 *gr.* and the difference of the sides 38 *gr.* 30 *m.* and 40 *gr.* being 1 *gr.* 30 *m.* the difference of their versed sines will be the same with the distance betweene the right sines of 20 *gr.* and 88 *gr.* 30 *m.* This difference I take, and make it a parallel sine of the lesser side 38 *gr.* 30 *m.* so the parallell Radius will be the fourth proportionall. Then coming to the second operation, I make this fourth proportionall a parallell sine of the greater side 40 *gr.* and take out his parallell Radius. For this measured from 90 *gr.* beyond the center in the lines of *sines* stretched forth at their full length, will be the versed sine of 130 *gr.* 3 *m.*

2 I may finde an angle by knowing three sides, by that which I haue elsewhere demonstrated vpon *Barth. Pitiscus*, and

and that at one operation in this maner.

As the sine of the greater side
is to the secant of the complement of the other side:
So the difference of sines of the complement of the base, and the arke compounded of the lesser side with the complement of the greater,
to the versed sine of the angle required.

So in the same triangle Z *P S*, hauing the same complements giuen, the angle at *P*, which sheweth the houre from the meridian, will be found as before 31 *gr.* 34 *m.*

For the sides being 38 *gr.* 30 *m.* and 70 *gr.* 0 *m.* I take the secant of the complement of 38 *gr.* 30 *m.* and make it a parallell sine of 70 *gr*; then keeping the Sector at this angle, I consider that the complement of 70 *gr.* being 20 *gr.* added vnto 38 *gr.* 30 *m.* the compounded side (which is here the meridian-altitude) will be 58 *gr.* 30 *m*; and that the base being 40 *gr.* the difference of sines of the compounded side and the complement of the base will be (as before) the distance betweene the sines of 50 *gr.* and 58 *gr.* 30 *m.* Wherefore I take out this difference, and lay it on both the lines of *sines* from the center: so the parallell taken in the termes of this difference, and measured from 90 *gr.* toward the center, doth giue the versed sine of 31 *gr.* 34 *m.*

The other angles *P* Z *S*, *P S* Z, may be found in the same sort; but hauing the sides and one angle, it will be sooner done by that which we shewed before in the 18. *Prop.*

20 *To find a side by knowing the three angles.*

If for the greater angle we take his complement to 180 *gr.* the angles shall be turned into sides, and the sides into angles, & the operation shall be the same, as in the former *Prop.*

21 *To finde a side, by hauing the other two sides, and the angle comprehended.*

This proposition being the conuerse of the nineteenth,

may be wrought accordingly; but the best way both for it and those which follow, is to resolue them into two rectangles, by letting downe a perpendicular, as was shewed in the first *Prop.*

So in the triangle Z *P S*, hauing Z *P* the complement of the latitude, and *PS* the complement of the declination, with Z *P S* the angle of the houre from the meridian, we may find Z *S* the complement of the altitude of the Sunne.

For hauing let downe the perpendicular Z *R* by the first *Prop.* we haue two triangles, Z *R P*, Z *R S*, both rectangle at *R*. Then may we find the side *PR*, either by the second, or tenth, or eleuenth *Prop*; which taken out of *P S*, leaueth the side *RS*: with this *R S* and Z *R* we may find the base Z *S* by the fourth *Prop.*

Or hauing let downe the perpendicular *SM*, we haue two rectangle triangles *S M Z*, *S M P*. Then may we find *M P*, from which if we take Z *P*, there remaineth *M* Z: but with *M* Z and *S M*, we may find the base Z *S*.

22 *To find a side, by hauing the other two sides, and one of the angles next the inquired side.*

So in the triangle *Z P S*, hauing Z *P* the complement of the latitude, and *P S* the complement of the declination, with *P* Z *S* the angle of the azimuth, we may finde Z *S* the complement of the altitude of the Sunne.

For hauing Z *P*, and the angle at Z, we may to *S* Z produced, let downe a perpendicular *P V*. Then we haue two rectangle triangles, P V Z, P V S, wherein if we find the sides V Z, V S, and take the one out of the other, there will remain the side inquired Z *S*.

23 *To find a side, by hauing one side, and the two angles next the inquired side.*

So in the triangle A B D, hauing AB the place of the sun, and *B A D* the angle of the greatest declination, and *A D B* the

the angle of the equator with the horizon, we may find *A D* the oblique ascention.

For hauing let downe *B C* the perpendicular of declination, we haue two rectangle triangles, *A C B*, *D C* B. Then may we find *A C* the right ascention, and D C the ascentionall difference; and comparing the one with the other, there remaineth *A D*.

24 *To find a side, by hauing two angles, and the side inclosed by them.*

So in the triangle *Z P S*, hauing the angles at Z and P, with the side intercepted Z P, we may find the side P S. For hauing let downe the perpendicular P V, we haue two rectangles P V Z, P V S. Then may we find the angle V P Z, eiby the seuenth, or fifteenth, or sixteenth *Prop.* which added to Z P S, maketh the angle V P S: with this V P S and P V, we may find the base PS, according to the 13 *Prop.*

25 *To find an angle by hauing the other two angles and the side inclosed by them.*

So in the triangle Z P *S*, hauing the angles at Z and P, with the side intercepted Z P, we may finde the other angle Z *S* P. For hauing let downe the perpendicular *Z R*, we haue two rectangles Z R P, Z R *S*. Then may we finde the angle P Z R by the sixteenth *Prop.* and that compared with P Z *S*, leaueth the angle R Z *S*: with this *R Z S* and *Z R* we may find the angle required *Z S R*, according to the sixth *Prop.*

26 *To finde an angle, by hauing the other two angles, and one of the sides next the inquired angle.*

So in the triangle A B D, hauing the angles at *A* and *D*, with the side A B, we may find the angle A B D. For hauing let downe the perpendicular BC, we haue two rectangles,

ACB, DCB. Then may we find the angles ABC, DBC, and take DBC out of ABC; for so there remaineth the angle required ABD.

27 *To finde an angle, by knowing two sides, and the angle contained by them.*

So in the triangle ZPS, hauing the sides ZP, PS, with the angle comprehended ZPS, we may find the angle PZS. For hauing let downe the perpendicular SM, we haue two rectangles *SMZ*, *S*MP. Then may we find the side MP, and taking ZP out of MP, there remaineth *MZ*: with this *MZ* and the perpendicular *MS*, we may finde the angle *MZS*, by the fourteenth *Prop.* This angle MZ*S*, taken out of 180 *gr.* there remaineth PZ*S*.

28 *To find an angle by knowing the two sides next it, and one of the other angles.*

So in the triangle *ZPS*, hauing the sides *ZP* and *PS*, with the angle *PZS*, we may find the angle Z*PS*. For hauing let downe the perpendicular *PV*, we haue two rectangles PVZ, PVS. Then may we find the angles VPZ, VPS; and taking VP*Z* out of VPS, there remaineth ZP*S*, which was required.

These 28 cases are all that can fall out in any sphericall triangle: if any do not presently vnderstand them, let them once more reade ouer the vse of the globes, and they shall soone become easie vnto them.

CHAP.

CHAP. VI.

Of the vse of the Meridian line in Nauigation.

THe *Meridian* line is here set on the side of the Sector, stretched forth at full length, on the same plane with the line of *lines* and *Solids*, and is diuided vnequally toward 87 *gr.* (whereof 70 *gr.* are about one halfe) in such sort as the Meridian in the cart of *Mercators* proiection. The vse of it may be

1 *To diuide a sea-chart according to* Mercators *proiection.*

If a degree of the equator on the sea-chart be equall to the hundred part of the line of *lines* in the *Sector*, the degrees of the *Meridian* vpon the *Sector*, shall giue the like degrees vpon the sea-chart: if otherwise they be vnequall, then may the meridians of the sea-chart be diuided in such sort as the line of *Meridians* is diuided on the *Sector*, by that which we shewed before in the 8. *Prop.* of the line of *lines*.

But to auoid error, I haue here set downe a Table, whereby the Meridian line may be diuided out of the degrees of the equator, supposing each degree to be subdiuided into a thousand parts. By which Table, & the vsuall Table of *Sines, Tangents* and *Secants*, the proportions following may be also resolued arithmetically. For the maner of diuision, let the equator (or one of the parallels if it be a particular chart) be drawne, and diuided, and crossed with parallell meridians, as in the common sea-chart: then looke into the Table, and let the distance of 40 *gr.* in the meridian, from the equator, be equall to 43 *gr.* 711 *parts* of the equator; let 50 *gr.* in the meridian from the equator, be equall to 57 *gr.* 909 *parts* of the equator; and so in the rest.

M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Par
0	0	0	3	3	001	6	6	011	9	9	037	12	12	088
		100		3	101		6	111		9	138		12	190
		200		3	201		6	212		9	239		12	293
		300		3	301		6	312		9	341		12	395
		400		3	402		6	413		9	442		12	497
		500		3	502		6	514		9	543		12	600
		600		3	602		6	614		9	645		12	702
		70		3	702		6	715		9	746		12	805
		800		3	803		6	816		9	848		12	907
		900		3	903		6	916		9	949		13	010
1	1	000	4	4	003	7	7	017	10	10	051	13	13	112
	1	100		4	103		7	118		10	152		13	215
	1	200		4	204		7	219		10	254		13	318
	1	300		4	304		7	319		10	355		13	421
	1	400		4	404		7	420		10	457		13	523
	1	500		4	504		7	521		10	559		13	626
	1	600		4	605		7	622		10	661		13	729
	1	700		4	705		7	723		10	762		13	832
	1	800		4	805		7	824		10	864		13	935
	1	900		4	906		7	925		10	966		14	038
2	2	000	5	5	006	8	8	026	11	11	068	14	14	141
	2	100		5	106		8	127		11	170		14	244
	2	200		5	207		8	228		11	272		14	347
	2	300		5	307		8	329		11	374		14	450
	2	400		5	408		8	430		11	476		14	553
	2	500		5	508		8	531		11	578		14	656
	2	601		5	609		8	632		11	680		14	760
	2	701		5	709		8	733		11	782		14	863
	2	801		5	810		8	834		11	884		14	967
	2	901		5	910		8	936		11	986		15	070
3	3	001	6	6	011	9	9	037	12	12	088	15	15	174

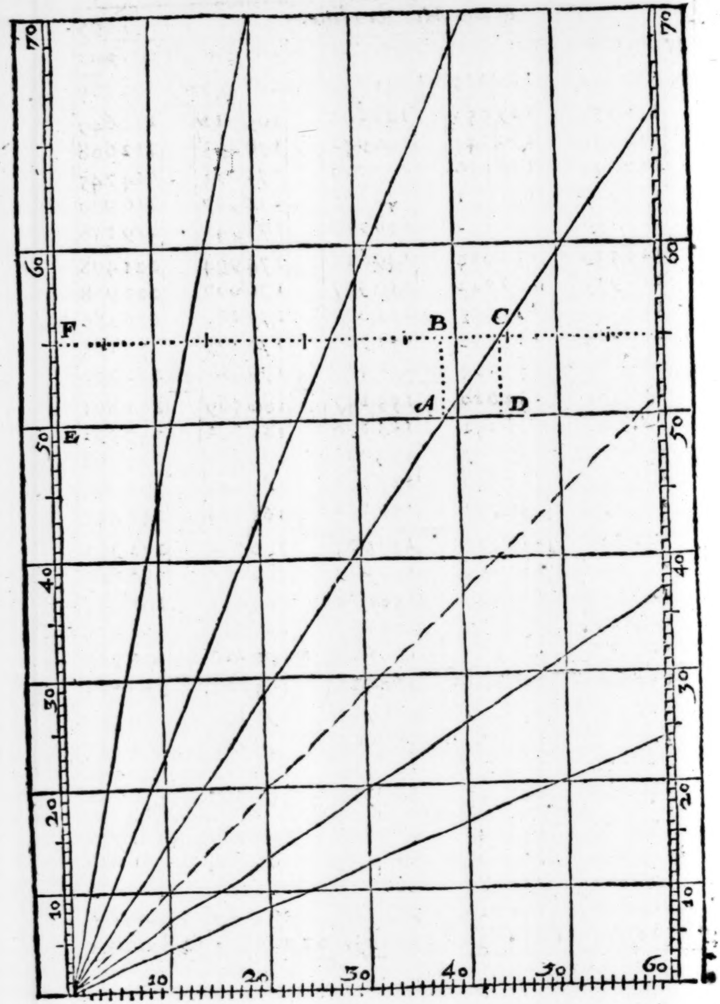
M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	part	M	G	part
15	15	174	18	18	303	21	21	486	24	24	734	27	28	058
	15	277		18	408		21	593		24	844		28	171
	15	381		18	513		21	701		24	953		28	283
	15	485		18	619		21	808		25	063		28	396
	15	588		18	724		21	915		25	173		28	508
	15	692		18	830		21	023		25	282		28	621
	15	796		18	935		22	130		25	392		28	734
	15	900		19	041		22	238		25	502		28	847
	16	004		19	146		22	345		25	613		28	959
	16	107		19	251		22	453		25	723		29	072
16	16	211	19	19	356	22	22	561	25	25	833	28	29	186
	16	316		19	463		22	669		25	943		29	299
	16	420		19	569		22	777		26	054		29	413
	16	524		19	675		22	885		26	164		29	526
	16	628		19	781		22	993		26	275		29	640
	16	732		19	887		23	101		26	386		29	753
	16	836		19	993		23	210		26	497		29	867
	16	941		20	100		23	318		26	608		29	981
	17	045		20	206		23	427		26	719		30	095
	17	150		20	312		23	535		26	830		30	300
17	17	255	20	20	419	23	23	643	26	26	941	29	30	324
	17	359		20	525		23	752		27	052		30	438
	17	464		20	632		23	861		27	164		30	553
	17	568		20	738		23	970		27	275		30	667
	17	673		20	845		24	079		27	387		30	782
	17	778		20	952		24	188		27	499		30	897
	17	883		21	059		24	297		27	610		31	012
	17	988		21	165		24	406		27	722		31	127
	18	093		21	272		24	515		27	834		31	242
	18	198		21	379		24	624		27	946		31	357
18	18	303	21	21	486	24	24	734	27	28	058	30	31	473

M	Gr	Par	M	G	Par	M	Gr	Par	M	Gr	Par	M	G	Par
30	31	473	33	34	992	36	38	633	39	42	415	42	46	362
	31	588		35	111		38	757		42	544		46	496
	31	704		35	231		38	880		42	673		46	631
	31	820		35	350		39	004		42	802		46	766
	31	936		35	470		39	129		42	931		46	902
	32	052		35	590		39	253		43	061		47	037
	32	168		35	710		39	377		43	191		47	173
	32	284		35	830		39	502		43	320		47	309
	32	40		35	950		39	627		43	451		47	445
	32	517		36	071		39	752		43	581		47	581
31	32	633	34	36	191	37	39	877	40	43	711	43	47	718
	32	750		36	312		40	002		43	842		47	855
	32	867		36	433		40	128		43	973		47	992
	32	984		36	554		40	253		44	104		48	129
	33	101		36	675		40	379		44	235		48	267
	33	218		36	796		40	505		44	366		48	404
	33	336		36	917		40	631		44	498		48	542
	33	453		37	039		40	757		44	630		48	681
	33	571		37	161		40	884		44	762		48	819
	33	688		37	283		41	011		44	894		48	958
32	33	806	35	37	405	38	41	137	41	45	026	44	49	097
	33	924		37	527		41	264		45	159		49	236
	34	042		37	649		41	392		45	292		49	375
	34	161		37	771		41	519		45	425		49	515
	34	279		37	894		41	646		45	558		49	655
	34	397		38	017		41	774		45	691		49	795
	34	516		38	140		41	902		45	825		49	935
	34	635		38	263		42	030		45	959		50	076
	34	754		38	386		42	158		46	093		50	217
	34	873		38	509		42	287		46	227		50	358
33	34	992	36	38	633	39	42	415	42	46	362	45	50	499

M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	part	M	Gr	part
45	50	499	48	54	860	51	59	481	54	64	412	57	69	711
	50	641		55	010		59	640		64	582		69	895
	50	783		55	160		59	800		64	753		70	080
	50	925		55	310		59	960		64	924		70	263
	51	068		55	460		60	120		65	096		70	449
	51	210		55	611		60	280		65	268		70	635
	51	353		55	762		60	441		65	440		70	821
	51	496		55	913		60	602		65	613		71	008
	51	639		56	065		60	763		65	786		71	195
	51	783		56	217		60	925		65	960		71	383
46	51	927	49	56	369	52	61	088	55	66	134	58	71	572
	52	071		56	522		61	250		66	308		71	761
	52	215		56	675		61	413		66	483		71	950
	52	360		56	828		61	577		56	659		72	140
	52	505		56	981		61	740		66	835		72	331
	52	650		57	135		61	904		67	011		72	522
	52	795		57	289		62	069		67	188		72	714
	52	941		57	444		62	234		67	365		72	906
	53	087		57	598		62	399		67	543		73	099
	53	233		57	754		62	564		67	721		73	292
47	53	380	50	57	909	53	62	730	56	67	900	59	73	486
	53	526		58	065		62	897		68	079		73	680
	53	673		58	221		63	063		68	258		73	875
	53	821		58	377		63	231		68	438		74	071
	53	968		58	534		63	398		68	618		74	267
	54	116		58	691		63	566		68	799		74	464
	54	264		58	848		63	734		68	981		74	661
	54	413		59	006		63	903		69	163		74	859
	54	562		59	164		64	072		69	345		75	057
	54	711		59	322		64	242		69	528		75	256
48	54	860	51	59	481	54	64	412	57	69	711	60	75	456

M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr.	Par
60	75	456	63	81	749	66	88	725	69	96	575	72	105	579
	75	656		81	970		88	971		96	854		105	904
	75	857		82	191		89	219		97	135		100	230
	76	059		82	413		89	467		97	418		106	558
	76	261		82	635		89	716		97	701		106	888
	76	464		82	860		89	967		97	986		107	220
	76	667		83	084		90	218		98	272		107	553
	76	871		83	310		90	470		98	560		107	888
	77	076		83	536		90	723		98	849		108	226
	77	281		83	763		90	978		99	139		108	565
61	77	487	64	83	990	67	91	232	70	99	431	73	108	906
	77	694		84	219		91	489		99	724		10,	249
	77	901		84	448		91	746		100	018		109	594
	78	109		84	678		92	005		100	314		109	941
	78	317		84	909		92	264		100	612		110	290
	78	526		85	141		92	525		100	910		110	641
	78	736		85	374		92	787		101	211		110	994
	78	947		85	607		93	050		101	513		111	349
	79	158		85	842		93	314		101	816		111	707
	79	370		86	077		93	579		102	121		112	066
62	79	583	65	86	313	68	93	846	71	102	427	74	112	428
	79	796		86	550		94	113		102	735		112	792
	80	010		86	788		94	382		103	044		113	158
	80	225		87	027		94	652		103	356		113	526
	80	441		87	267		94	923		103	668		113	897
	80	657		87	508		95	195		103	983		114	270
	80	874		87	749		95	468		104	299		114	645
	81	091		87	992		95	743		104	616		115	023
	81	310		88	235		96	019		104	936		115	403
	81	529		88	480		96	296		105	257		115	786
63	81	749	66	88	725	69	96	575	72	105	579	75	116	171

M	Gr.	Par	M	Gr.	Par	M	Gr.	Par	M	Gr.	Par	M	Gr.	part
75	116	171	78	129	075	81	145	650	84	168	947	87	208	705
	116	559		129	558		146	292		169	912		210	649
	116	949		130	045		146	942		170	893		212	668
	117	342		130	536		147	600		171	891		214	745
	117	737		131	031		148	265		172	907		216	909
	118	135		131	530		148	937		173	941		219	158
	118	536		132	034		149	618		174	994		221	498
	118	939		132	542		150	307		176	067		223	938
	119	345		133	055		151	003		177	160		226	486
	119	755		133	572		151	709		178	275		229	153
76	120	160	79	134	094	82	152	423	85	179	411	88	231	950
	120	581		134	620		153	147		180	569		234	891
	121	003		135	151		153	878		181	752		237	991
	121	420		135	687		154	620		182	960		241	268
	121	843		136	228		155	372		184	194		244	744
	122	270		136	775		156	132		185	454		248	445
	122	700		137	326		156	903		186	743		252	402
	123	133		137	383		157	685		188	062		256	652
	123	570		138	445		158	478		189	411		261	243
	124	009		139	012		159	281		190	793		266	235
77	124	452	80	139	585	83	160	096	86	192	210	89	271	705
	124	898		140	164		160	922		193	661		277	753
	125	348		140	748		161	761		195	151		284	517
	125	801		141	339		162	612		196	680		292	191
	126	258		141	936		163	475		198	251		301	058
	126	718		142	538		164	352		199	867		311	563
	127	182		143	147		165	242		201	529		324	455
	127	649		143	763		166	146		203	240		341	166
	128	121		144	385		267	065		205	005		365	039
	128	596		145	014		167	999		206	825		408	011
78	129	075	81	145	650	84	168	947	87	208	705	90	Infinite	



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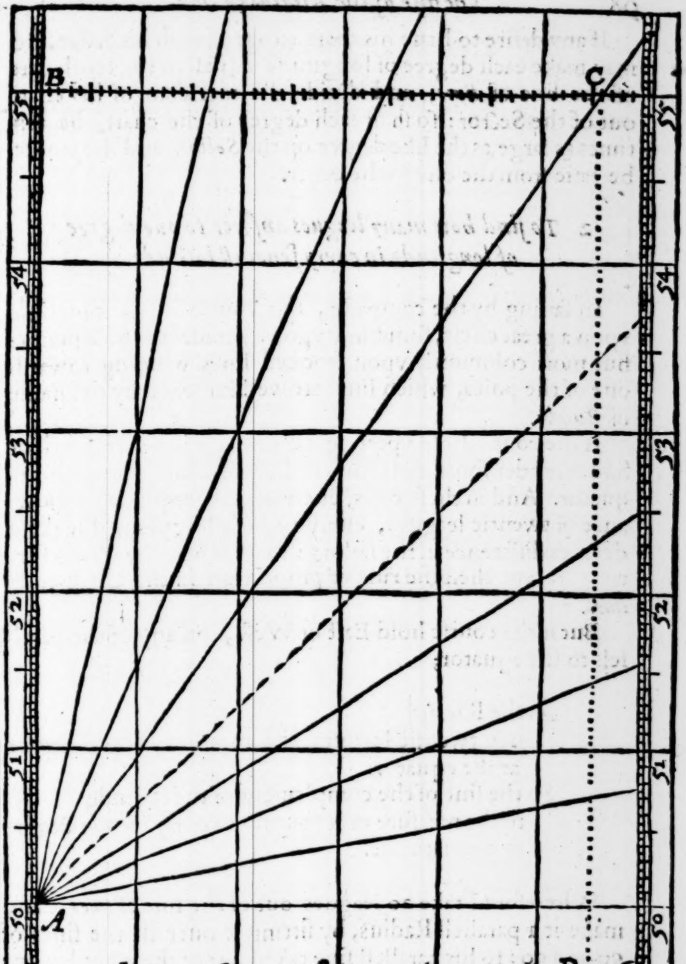
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If any desire to haue his chart to agree with his *Sector*, he may make each degree of longitude equall to the tenth part of the line of *lines*, and diuide the meridian of his chart out of the Sector: so shall each degree of the chart, be ten times as large as the like degree on the *Sector*, and the worke be easie from the one to the other.

2 *To find how many leagues answer to one degree of longitude in euery seuerall latitude.*

In sailing by the compasse, the course holds sometime vpon a great circle, sometime vpon a parallel to the equator; but most commonly vpon crooked lines winding towards one of the poles, which lines are well knowne by the name of *Rumbs*.

If the course hold vpon a great circle, it is either North or South, vnder some meridian, or East or West, vnder the equator. And in these cases, euery degree requires an allowance of twentie leagues, euery twentie leagues will make a degrees difference in the sailing: so that here needs no further precept then the rule of proportion in the Chapter of *lines*.

But if the course hold East or West, on any of the parallels to the equator;

As the Radius
is to twentie leagues, the measure of one degree at the equator:
So the sine of the complement of the latitude
to the measure of leagues answering to one degree in that latitude.

Wherefore I take 20 leagues out of the line of *lines*, and make it a parallell Radius, by fitting it ouer in the sines of 90 and 90: so his parallell sine taken out of the complement of the latitude, and measured in the line of *lines*, shall shew the number of leagues required.

Thus

Thus in the latitude of 18 *gr.* 12 *m.* we shall find 19 leagues answering to one degree of longitude, and 18 leagues in the latitude of 25 *gr.* 15 *m.* and as in this Table.

Gr.	1.	Lg.
0	0	20
18	12	19
25	15	18
31	48	17
36	52	16
41	25	15
45	34	14
49	28	13
53	8	12
56	38	11
60	0	10
63	15	9
66	25	8
69	30	7
72	32	6
75	31	5
78	28	4
81	23	3
84	15	2
87	8	1

This may be done more readily without opening the Sector, by doubling the sine of the complement of the latitude, as may appeare in the same example.

It may also be done by the line of meridians, either vpon the Sector, or vpon the chart. For if we open a paire of compasses to the quantitie of one degree of longitude in the equator, and measure it in the meridian line, setting one foot as much aboue the latitude giuen, as the other falleth beneath it, so that the latitude may be in the middle betweene the feete of the compasses, the number of leagues intercepted shall be that which was required.

But if the course hold vpon any of the *rumbs*, betweene a parallell of the equator and the meridian, we are to consider besides the quarter of the world to which we tend, which must be alwayes knowne.

1 The difference of longitude at least in generall.
2 The difference of latitude, and that in particular.
3 The *rumb* whereon the course holds.
4 The distance vpō the *rumb*, which is the distance, which we are here to consider, and is alwayes somewhat greater then the like distance vpon a greater circle. And for these first I shew in generall this third *Prop.*

3 *To finde how many leagues do answer to one degree of latitude in euery seuerall Rumb.*

As the sine of the complement of the *rumb* frō the meridian, is to 20 leagues the measure of one degree at the meridian: So the Radius
to the leagues answering to one degree vpon the Rumb.

Wherfore I take 20 leagues out of the line of *lines*, and make it a parallell sine of the complement of the Rumb from the meridian; so his parallell Radius taken and measured in the line of *lines*, shall shew the number of leagues required.

Thus in the first Rumb from the meridian, we shall finde 20 *lgs* 39 *parts* answering to one degree of latitude, and 21 *lgs* 65 *parts* in the second Rumb, &c. as in this Table, where we subdiuide each league into a hundred parts, and shew besides what inclination the rumb hath to the meridian.

This may be done more readily without opening the *Sector*, by doubling the secant of the latitude, as may appeare in the same example.

It may also be done vpon the chart, if we take the distance vpon the Rumb betweene two parallels, and measure it in the meridian line, as farre aboue the greater latitude as beneath the lesser. For so the number of leagues intercepted, shall be that which was required.

This considered in generall, I shew more particularly in twelue *Prop.* following, how of these foure any two being giuen, the other two may be found, both by *Mercators* chart, and by this *Sector*.

Rumbs.	*Inclination to the meridian*		*Number of leags.*	
	Gr.	*Mi.*	*Lgs*	*Par*
	2	49	20	02
	5	37	20	10
	8	26	20	22
1	11	15	20	39
	14	4	20	62
	16	52	20	90
	19	41	21	24
2	22	30	21	65
	25	19	22	12
	28	7	22	68
	30	56	23	32
3	33	45	24	05
	36	34	24	90
	39	22	25	87
	42	11	26	99
4	45	0	28	28
	47	49	29	78
	50	37	31	52
	53	26	33	57
5	56	15	36	00
	59	4	38	90
	61	52	42	43
	64	41	36	78
6	67	30	52	26
	70	19	59	37
	73	7	68	90
	75	56	82	31
7	78	45	102	52
	81	34	136	30
	84	22	205	24
	87	11	407	60
8	90	0	*Infinita.*	

1 *By one latitude Rumb and distance to find the difference of latitudes.*

As the Radius
to the sine of the complement of the Rumb from the meridian:
So the distance vpon the Rumb,
to the difference of latitudes.

Let the place giuen be *A* in the latitude of 50 *gr.* *C* in a greater latitude, but vnknowne, the distance vpon the Rumb being 6 *gr.* betweene them, and the Rumb the third from the meridian.

First I take 6 *gr.* for the distance vpon the Rumb, out of the line of *lines*, and make it a parallell Radius, by putting it ouer in the sines of 90 and 90. Then keeping the *Sector* at this angle, I take out the parallell sine of 56 *gr.* 15 *m.* which is the sine of the complement of the third Rumb from the meridian, and measuring it in the line of *lines*, I find it to be 5 *gr.* and such is the difference of latitude required.

Or I may take out the sine of 56 *gr.* 15 *m.* for the complement of the third Rumb from the meridian, make it a parallell Radius; then keeping the *Sector* at this angle, I take 6 *gr.* for the distance, either out of the line of *lines*, or any other scale of equall parts, or else out of the meridian line, and lay it on both sides of the *Sector* from the center, either on the line of *lines* or *sines*: so the parallell taken from the termes of this distance, and measured in the same scale wherein the distance was measured, shall shew the difference of latitude to be 5 *gr.* as before.

But in shorter distances, such as fall within the compasse of a dayes sailing, this worke will hold much better. As may appeare by comparing the worke with the Table following: where the numbers in the front do signifie the leagues; those in the side, the Rumb; and the rest in the middle, the difference of latitude.

Lgs	100		80		60		40		20	19	18	17	16	15
Rum.	G.	M	G.	M	G.	M	G.	M	M	M	M	M	M	M
	5	0	4	0	3	0	2	0	60	57	54	51	48	45
	4	59	3	59	2	59	1	59	60	57	54	51	48	45
	4	58	3	58	2	59	1	59	60	57	54	51	48	45
	4	56	3	57	2	58	1	58	59	56	53	50	47	44
1	4	54	3	55	2	56	1	57	59	56	53	50	47	44
	4	51	3	53	2	55	1	56	58	56	52	50	47	43
	4	47	3	50	2	52	1	55	57	55	52	49	46	43
	4	42	3	46	2	49	1	53	56	54	51	48	45	42
2	4	37	3	42	2	46	1	51	55	53	50	47	44	41
	4	31	3	37	2	43	1	48	54	52	49	46	43	40
	4	25	3	32	2	39	1	46	53	50	48	45	42	39
	4	17	3	26	2	34	1	43	51	49	46	44	41	38
3	4	10	3	20	2	30	1	40	50	47	45	42	40	37
	4	1	3	13	2	25	1	36	48	46	43	41	39	36
	3	52	3	5	2	19	1	32	46	44	42	39	37	35
	3	42	2	58	2	13	1	28	44	42	40	38	36	33
4	3	32	2	50	2	7	1	25	42	40	38	36	34	32
	3	22	2	41	2	1	1	21	40	38	36	34	32	30
	3	10	2	32	1	54	1	16	38	36	34	32	30	28
	2	59	2	23	1	47	1	12	36	34	32	30	29	27
5	2	47	2	14	1	40	1	7	33	32	30	28	27	25
	2	34	2	3	1	32	1	2	31	29	28	26	25	23
	2	22	1	53	1	25	0	57	28	27	25	24	23	22
	2	8	1	43	1	17	0	52	26	24	23	22	21	19
6	1	55	1	32	1	8	0	46	23	22	21	20	18	17
	1	41	1	20	1	0	0	40	20	19	18	17	16	15
	1	27	1	9	0	52	0	35	17	16	16	15	14	13
	1	13	0	58	0	44	0	30	15	14	13	12	12	11
7	0	59	0	47	0	35	0	24	12	11	11	10	9	9
	0	44	0	36	0	26	0	18	9	8	8	7	7	7
	0	30	0	24	0	18	0	12	6	6	5	5	5	4
	0	15	0	12	0	9	0	9	3	3	3	3	2	2
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0

14	13	12	11	10	9	8	7	6	5	4	3	2	1	Lg
M	M	M	M	M	M	M	M	M	M	M	M	M	M	Run.
40	39	36	33	30	27	24	21	18	15	12	9	6	3	
42	39	36	33	30	27	24	21	18	15	12	9	6	3	
42	39	36	33	30	27	24	21	18	15	12	9	6	3	
42	39	36	33	30	27	24	21	18	15	12	9	6	3	
41	38	35	32	29	26	24	21	18	15	12	9	6	3	1
41	38	35	32	29	26	23	20	17	15	12	9	6	3	
40	37	34	32	29	26	23	20	17	14	11	9	6	3	
40	37	34	31	28	25	23	20	17	14	11	8	6	3	
39	36	33	31	28	25	22	19	17	14	11	8	6	3	2
38	35	33	30	27	24	22	19	16	14	11	8	5	3	
37	34	32	29	26	24	21	19	16	13	11	8	5	3	
36	33	31	28	25	23	21	18	15	13	10	8	5	3	
35	32	30	27	25	22	20	17	15	12	10	7	5	2	3
34	30	29	26	24	22	19	17	14	12	10	7	5	2	
33	30	28	25	23	21	19	16	14	12	9	7	5	2	
31	29	27	24	22	20	18	16	13	11	9	7	4	2	
30	28	25	23	21	19	17	15	13	11	8	6	4	2	4
28	26	24	22	20	18	16	14	12	10	8	6	4	2	
27	25	23	21	19	17	15	13	11	10	8	6	4	2	
25	23	21	20	18	16	14	13	11	9	7	5	4	2	
23	22	20	18	17	15	13	12	10	8	7	5	3	2	5
22	20	18	17	15	14	12	11	9	8	6	5	3	2	
20	18	17	16	14	13	11	10	8	7	6	4	3	1	
18	17	15	14	13	12	10	9	8	6	5	4	3	1	
16	15	14	13	11	10	9	8	7	6	5	3	2	1	6
14	13	12	11	10	9	8	7	6	5	4	3	2	1	
12	11	10	10	9	8	7	6	5	4	3	3	2	1	
10	9	9	8	7	7	6	5	4	4	3	2	1	1	
8	8	7	6	6	6	5	4	3	3	2	2	1	1	7
6	6	5	5	4	5	4	3	3	2	2	1	1	1	
4	4	4	3	3	3	2	2	2	1	1	1	1	0	
2	2	2	2	1	1	1	1	1	1	1	1	c	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	8

In the Chart let a meridian *A B* be drawne through *A*, and in *A* with *A* B make an angle of the Rumb *B A C*. Then open the compasses, accrding to the latitude of the places, to *E F* the quantitie of *6 gr.* in the meridian, transferring them into the Rumb from *A* to *C*, and through *C* draw the parallell *B C*, crossing the meridian *A B* in *B*: so the degrees in the meridian from *A* to *B*, shall shew the difference of latitude to be *5 gr.*

2 *By the Rumb and both latitudes to find the distance vpon the Rumb.*

As the sine of the complement of the Rumb from the meri-(dian,
is to the Radius:
So the difference of latitudes,
to the distance vpon the Rumb.

As if the places giuen were *A* in the latitude of *50 gr.* *C* in the latitude of *55 gr.* and the Rumb the third from the meridian.

Here I may take *5 gr.* for the difference of latitude out of the line of *lines*, and put it ouer in the sine of *56 gr. 15 m.* for the complement of the third Rumb from the meridian. Then keeping the *Sector* at this angle, I take out the parallell Radius, and measuring it in the line of *lines*, I find it to be *6 gr.* and such is the distance vpon the Rumb, which was required.

Or I may take the laterall Radius, and make it a parallell sine of *56 gr. 15 m.* the complement of the Rumb from the meridian: then keeping the Sector at this angle, I take *5 gr.* for the difference of latitude, either out of the line of *lines*, or out of some other scale of equall parts, and lay it on both sides of the *Sector* from the center, either on the line of *lines* or of *sines*: so the parallell taken from the termes of this difference, and measured in the same scale with the difference, shall shew the distance vpon the Rumb to be *6 gr.* or 120 leagues.

Or

Or keeping the *Sector* at this angle, I may take the difference betweene 50 *gr.* and 55 *gr.* out of the *Meridian* line, and measuring it in the equator, I shal find it to be equall to 8 *gr.* 22 *p.* of the equator. Wherefore I take the parallell betweene 822 and 822 out of the line of *lines*, and measuring it in the line of *lines* I shall find it to be 989; which shewes that according to this proiection, the distance vpon this third Rumb, answerable to the former distance of latitudes, will be equall to 9 *gr.* 89 *p.* of the equator.

Or the Sector remaining at this angle, I may take the difference betweene 50 *gr.* and 55 *gr.* out of the *Meridian* line, and lay it from the center on both sides of the *Sector*, either on the line of *lines* or of *sines*: so the parallell taken from the termes of this difference, shall be the very line of distance required, the same with *A C* or *E F* vpon the chart; which may serue for the better pricking downe of the distance vpon the Rumb, without taking it forth of the *Meridian* line, as in the former *Prop.*

Or if the Rumb fall nearer to the equator, that the laterall Radius cannot be fitted ouer in it, this proposition may be wrought by parallell entrance.

For if I first take out the sine of 56 *gr.* 15 *m.* and make it a parallell Radius, by fitting it ouer in the sines of 90 and 90, or in the ends of the lines of *lines*, and then take 5 *gr.* for the difference of latitudes out of the line of *lines*, and carrie it parallell to the former, I shall find it to crosse both lines of *lines* in the points of 6: and so it giues the same distance as before.

Or if the distance be small, it may be found by the former Table. For the Rumb being found in the side of the Table, and the difference of latitude in the same line; the top of the columne wherein the difference of latitude was found, shall giue the number of leagues in the distance required.

Or we may finde this distance in the Table of Rumbs in the fift *Prop.* following. For according to the example looke into the Table of the third Rumb for 5 *gr.* of latitude, and there we shall finde 6 *gr.* 01 *parts* vnder the title of distance.

So if the difference of latitude vpon the same Rumb were 50 *gr.* the distance would be 60 *gr.* 13 *parts.* If the difference of latitude vpon the same Rumb were onely $\frac{1}{2}$ of a degree, the distance would be onely 60 *parts*, such as 100 doe make a degree.

In the chart let a Meridian *A B* be drawne through *A*, and parallels of latitude through *A* and *C*; & then in *A* with *A B* make an angle of the Rumb *B A C*: so the distance taken from *A* to *C*, and measured in the Meridian line, according to the latitude of the places, shall be found to be 6 *gr.* or 120 leagues. And such is the distance required.

3 *By the distance and both latitudes to find the Rumb.*

As the distance vpon the Rumb,
to the difference of latitudes:
So is the Radius
to the sine of the complement of the Rumb from the Meridian.

As if the places giuen were *A* in the latitude of 50 *gr.* *C* in the latitude of 55 *gr.* the distance betweene them being 6 *gr.* vpon the Rumb. First I take 6 *gr.* for the distance vpon the Rumb, and lay it on both sides of the *Sector* from the center; then out of the same scale I take 5 *gr.* for the difference of latitude, and to it open the *Sector* in the termes of the former distance: so the parallell Radius taken and measured in the *sines*, doth giue 56 *gr.* 15 *m.* the complement whereof 33 *gr.* 45 *m.* is the angle of the Rumbs inclination to the Meridian, which was required.

In the chart let a meridian *A B* be drawne through *A*, and parallels of latitude both through *A* and *C*; then open the compasses according to the latitude of the places to *E F* the quantitie of 6 *gr.* in the meridian, and setting one foote in *A*, turne the other till it crosse the parallell *B C* in *C*, and draw the right line *A C*: so the angle *B A C* shall shew the inclination of the Rumb to the Meridian to be 33 *gr.* 45 *m.* as before.

These

These three last *Prop.* depend one on the other, and may be wrought as truly by the common sea-chart as by this of *Mercators* proiection: and therefore in working them by the *Sector*, the distance and the difference of latitudes may as well or better be taken out of the line of *lines* (which here representeth the equator) or any other line of equall parts, as out of the inlarged degrees in the *meridian* line. But in the propositions following, the difference of longitude must be taken out of the equator; the difference of latitudes and distance vpon the Rumb, must alwayes be taken out of the *meridian* line; which I therefore call the proper difference, and proper distance.

4 *By the longitude and latitude of two places to find the Rumb.*

As if the places giuen were *A* in the latitude of 50 *gr.* *C* in the latitude of 55 *gr.* and the difference of longitude betweene them were 5 *gr.* 30 *m.*

In the chart let meridians and parallels be drawn through *A* and *C*, and a straight line for the Rumb from *A* to *C*; then by that we shewed *Cap.2.Prop.9.* inquire the quantitie of the angle *B A C*, and it shall be found to be 33 *gr.* 45 *m.* which is the third Rumb from the Meridian. Wherefore the proportion holds for the *Sector*,

As *A B* the proper difference of latitude,
is to *B C* the difference of longitude:
So *A B* as Radius,
to *B C* the tangent of the Rumb from the Meridian.

According to this I take the proper difference of latitude from 50 *gr.* to 55 *gr.* out of the line of *meridians*, and lay it on both sides of the *Sector* from the center; then I take the difference of longitude 5 *gr.* $\frac{1}{2}$ out of the line of *lines*, and to it open the *Sector* in the termes of the former difference of latitudes: so the parallell Radius taken from betweene 90 and 90, and measured in the greater *tangent* on the side of the *Se-*

ctor, doth giue 33 *gr.* 45 *m.* for the Rumb required.

But if the Rumb fall nearer to the equator;

As *A D* the difference of longitudes,
is to *D C* the proper difference of latitudes:
So *A D* as Radius,
to *D C* the tangent of the rumb from the equator.

According to this I take the former difference of latitudes from 50 *gr.* to 55 *gr.* out of the line of *Meridians*, and to it open the *Sector* in the termes of the difference of longitude reckoned in the line of *lines* from the center: so the parallell Radius taken and measured in the *tangent*, doth giue 56 *gr.* 15 *m.* for the rumb from the equator; which is the complement to the former 33 *gr.* 45 *m:* and so both wayes it is found to be the third rumb from the Meridian.

But if this rumb were to be found in the common sea-chart, it should seeme to be aboue 47 *gr.* which is more then the fourth rumb from the meridian.

5 *By the Rumb and both latitudes to find the difference of longitude.*

As if the places giuen were *A* in the latitude of 50 *gr.* and *C* in the latitude of 55 *gr.* and the rumb the third from the meridian.

In the chart, let a meridian be drawne through *A*, and a parallell of latitude through *C*; then in *A* with *A B* make the angle of the rumb from the meridian *B A C*, (as was shewed *Cap.2.Prop.10.*) So the degrees in the parallell betweene B and *C*, shall be found to be 5 *gr.* $\frac{1}{2}$, the difference of longitude which was required. Wherefore the proportion holds for the *Sector*.

As *A B* the Radius,
to B C the tangent of the rumb from the meridian:
So *A B* as proper difference of the latitudes,
to *B C* the difference of longitude.

According

According to this we may take the tangent of the Rumb which is here 33 *gr*.45 *m*.from the meridian, out of the greater *tangent* on the side of the *Sector*; and putting it ouer betweene 90 and 90, make it a Radius: then keeping the *Sector* at this angle, take the proper difference of latitudes from 50 *gr*.to 55 *gr*.out of the line of *Meridians*, and lay it on both sides of the *Sector* from the center: so the parallel taken from the termes of this difference, and measured in the line of *lines*, shall shew the difference of longitude to be 5 *gr*.$\frac{1}{2}$.

Or if the Rumb fall nearer the equator.

As *D C* the tangent of the *R*umb from the equator,
to *A D* the *R*adius:
So *D C* as proper difference of the latitudes,
to *A D* the difference of longitude.

According to this we may best work by parallell entrance, first taking 56 *gr*.15 *m*.for the angle of the *R*umb from the equator, out of the greater *tangent*, and make it a parallell *R*adius: then take the proper difference of latitudes out of the line of *meridians*, and carrie it parallell to the former: so we shall find it to crosse the line of *lines* in 5 *gr*.$\frac{1}{2}$. And this is the difference of longitude required, the same as before.

But if this difference were to be found by the common sea-chart, it should seeme to be only 3 *gr*.20 *m*.which is more then 2 *gr*.lesse then the truth. And yet this error would be greater, if either the latitude be greater, or the *R*umb fall nearer the equator: as may appeare by comparing the common sea-chart with the Tables following.

The first Rumbe from the Meridian.					*North and by East, South and by East,*					*North and by West, South and by West,*				
La	Long.		Dist.		La	Long.		Dist.		La	Long.		Dist.	
Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P.	Gr.	P.
0		0		0	30	6	26	30	59	60	15	01	61	18
1		20	1	02	31	6	49	31	61	61	15	41	62	20
2		40	2	04	32	6	72	32	63	62	15	83	63	21
3		60	3	06	33	6	96	33	65	63	16	26	64	23
4		80	4	08	34	7	20	34	67	64	16	71	65	25
5	1	00	5	10	35	7	44	35	69	65	17	17	66	27
6	1	20	6	12	36	7	68	36	71	66	17	65	67	29
7	1	40	7	14	37	7	92	37	73	67	18	15	68	31
8	1	60	8	16	38	8	17	38	75	68	18	67	69	33
9	1	80	9	18	39	8	43	39	77	69	19	21	70	35
10	2	00	10	20	40	8	70	40	78	70	19	78	71	37
11	2	20	11	22	41	8	96	41	80	71	20	37	72	39
12	2	40	12	24	42	9	22	42	82	72	21	00	73	41
13	2	61	13	25	43	9	50	43	84	73	21	66	74	43
14	2	81	14	27	44	9	76	44	86	74	22	36	75	45
15	3	02	15	29	45	10	04	45	88	75	23	10	76	47
16	3	22	16	31	46	10	33	46	90	76	23	90	77	49
17	3	43	17	33	47	10	62	47	92	77	24	75	78	51
18	3	64	18	35	48	10	91	48	94	78	25	67	79	53
19	3	85	19	37	49	11	21	49	96	79	26	67	80	55
20	4	06	20	39	50	11	52	50	98	80	27	76	81	57
21	4	27	21	41	51	11	83	52	0	81	28	97	82	59
22	4	49	22	43	52	12	15	53	2	82	30	32	83	61
23	4	70	23	45	53	12	47	54	4	83	31	84	84	63
24	4	92	24	47	54	12	81	55	6	84	33	61	85	62
25	5	14	25	49	55	13	16	56	8	85	35	69	86	67
26	5	36	26	51	56	13	50	57	10	86	38	24	87	69
27	5	58	27	53	57	13	86	58	12	87	41	52	88	71
28	5	80	28	55	58	14	23	59	14	88	46	15	89	73
29	6	03	29	57	59	14	62	60	16	89	54	06	90	75
30	6	26	30	59	60	15	01	61	18	90				

The second Rumbe from the Meridian.					North North-east. South South-east.					North North-west South South-west				
La	Long.		Dist.		La	Long		Dist.		La	Long.		Dist.	
Gr	Gr.	P.	Gr.	P	Gr	Gr.	P	Gr.	P.	Gr	Gr.	P.	Gr.	P.
0		0		0	30	13	03	32	47	60	31	25	64	94
1	0	42	1	08	31	13	51	33	55	61	32	09	66	03
2	0	83	2	16	32	14	00	34	64	62	32	96	67	11
3	1	24	3	25	33	14	49	35	72	63	33	86	68	19
4	1	65	4	33	34	15	00	36	80	64	34	79	69	27
5	2	07	5	41	35	15	50	37	88	65	35	71	70	35
6	2	49	6	49	36	16	00	38	97	66	36	75	71	44
7	2	91	7	57	37	16	51	40	05	67	37	80	72	52
8	3	32	8	66	38	17	03	41	13	68	38	88	73	60
9	3	74	9	74	39	17	56	42	21	69	40	00	74	68
10	4	16	10	82	40	18	10	43	30	70	41	19	75	77
11	4	59	11	90	41	18	65	44	38	71	42	43	76	85
12	5	01	12	99	42	19	20	45	46	72	43	74	77	93
13	5	43	14	07	43	19	76	46	54	73	45	11	79	01
14	5	85	15	15	44	20	33	47	62	74	46	57	80	10
15	6	28	16	23	45	20	92	48	71	75	48	12	81	18
16	6	71	17	32	46	21	50	49	79	76	49	78	82	26
17	7	14	18	40	47	22	11	50	87	77	51	55	83	34
18	7	58	19	48	48	22	72	51	95	78	53	46	84	42
19	8	01	20	56	49	23	35	53	03	79	55	54	85	51
20	8	45	21	65	50	23	98	54	12	80	57	82	86	59
21	8	90	22	73	51	24	63	55	20	81	60	33	87	67
22	9	34	23	81	52	25	30	56	28	82	63	13	88	76
23	9	79	24	89	53	25	98	57	37	83	66	32	89	84
24	10	24	25	98	54	26	68	58	45	84	69	99	90	92
25	10	70	27	06	55	27	39	59	53	85	74	32	92	00
26	11	16	28	14	56	28	12	60	61	80	79	63	93	09
27	11	62	29	22	57	28	87	61	70	87	86	46	94	17
28	12	08	30	31	58	29	64	62	78	88	96	10	95	25
29	12	55	31	39	59	30	44	63	86	89	112	57	96	33
30	13	03	32	47	60	31	25	64	94	90				

The third Rumbe from the Meridian.					North-east by North, South-east by South,					North-west by North, South-west by South.				
La	Long.		Dist.		La	Long.		Dist.		La	Long.		Dist.	
Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P.	Gr.	P
0		0		0	30	21	03	36	08	60	50	42	72	16
1	0	66	1	20	31	21	80	37	28	61	51	78	73	36
2	1	33	2	40	32	22	58	38	49	62	53	18	74	56
3	2	00	3	61	33	23	38	39	69	63	54	63	75	77
4	2	67	4	81	34	24	18	40	89	64	56	12	76	97
5	3	34	6	01	35	25	00	42	09	65	57	68	78	17
6	4	01	7	22	36	25	82	43	30	66	59	29	79	37
7	4	68	8	42	37	26	64	44	50	67	60	96	80	58
8	5	36	9	62	38	27	48	45	70	68	62	71	81	78
9	6	03	10	82	39	28	34	46	90	69	64	53	82	98
10	6	71	12	03	40	29	21	48	11	70	66	44	84	19
11	7	39	13	23	41	30	09	49	31	71	68	45	85	39
12	8	07	14	43	42	30	98	50	51	72	70	55	86	59
13	8	76	15	64	43	31	88	51	71	73	72	77	87	79
14	9	44	16	84	44	32	80	52	92	74	75	12	89	00
15	10	13	18	04	45	33	74	54	12	75	77	62	90	20
16	10	83	19	24	46	34	69	55	32	76	80	30	91	40
17	11	53	20	45	47	35	67	56	52	77	83	15	92	61
18	12	23	21	65	48	36	66	57	73	78	86	25	93	81
19	12	93	22	85	49	37	67	58	93	79	89	60	95	01
20	13	64	24	05	50	38	69	60	13	80	93	27	96	22
21	14	35	25	26	51	39	74	61	33	81	97	32	97	42
22	15	07	26	46	52	40	82	62	54	82	101	85	98	62
23	15	80	27	66	53	41	91	63	74	83	106	97	99	82
24	16	53	28	86	54	43	03	64	94	84	112	90	101	03
25	17	26	30	07	55	44	19	66	15	85	119	90	102	23
26	18	00	31	27	56	45	37	67	35	86	128	45	103	43
27	18	75	32	47	57	46	58	68	55	87	139	47	104	64
28	19	50	33	67	58	47	82	69	75	88	155	00	105	84
29	20	26	34	88	59	49	11	70	96	89	181	58	107	04
30	21	03	36	08	60	50	42	72	16	90				

The fourth Rumbe from the Meridian.					*North-east, South-east,*					*North-west, South-west.*				
La	Long.		Dist.		La	Long.		Dist.		La	Long.		Dist.	
Gr	Gr.	P	Gr.	P.	Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P.	Gr.	P.
0		0		0	30	31	47	42	43	60	75	46	84	85
1	1	00	1	41	31	32	63	43	84	61	77	49	86	27
2	2	00	2	83	32	33	81	45	25	62	79	58	87	68
3	3	00	4	24	33	34	99	46	67	63	81	75	89	09
4	4	00	5	66	34	36	19	48	07	64	83	99	90	51
5	5	01	7	07	35	37	41	49	50	65	86	31	91	92
6	6	01	8	49	36	38	63	50	91	66	88	73	93	34
7	7	02	9	90	37	39	88	52	33	67	91	23	94	75
8	8	03	11	31	38	41	14	53	74	68	93	85	96	17
9	9	04	12	73	39	42	42	55	15	69	96	58	97	58
10	10	05	14	14	40	43	71	56	57	70	99	43	98	99
11	11	07	15	56	41	45	03	57	98	71	102	43	100	41
12	12	09	16	97	42	46	36	59	40	72	105	58	101	82
13	13	11	18	38	43	47	72	60	81	73	108	91	103	24
14	14	14	19	80	44	49	10	62	22	74	112	43	104	65
15	15	17	21	21	45	50	50	63	64	75	116	17	106	06
16	16	21	22	63	46	51	93	65	05	76	120	17	107	48
17	17	25	24	04	47	53	38	66	46	77	124	45	108	89
18	18	30	25	45	48	54	86	67	88	78	129	08	110	31
19	19	36	26	87	49	56	37	69	29	79	134	10	111	72
20	20	42	28	28	50	57	91	70	71	80	139	59	113	14
21	21	49	29	70	51	59	48	72	12	81	145	65	114	55
22	22	56	31	11	52	61	09	73	54	82	152	42	115	96
23	23	64	32	53	53	62	73	74	95	83	160	10	117	38
24	24	73	33	94	54	64	41	76	37	84	168	95	118	79
25	25	28	35	35	55	66	13	77	78	85	179	41	120	21
26	26	94	36	77	56	67	90	79	20	86	192	21	121	62
27	28	06	38	18	57	69	71	80	61	87	208	71	123	04
28	29	18	39	60	58	71	57	82	02	88	231	95	124	45
29	30	32	41	01	59	73	49	83	44	89	271	71	125	86
30	31	47	42	43	60	75	46	84	85	90				

The fift Rumb be from the Meridian.					North-east and by East, South-east and by East,					North-west and by West, South-west and by West,				
La	Long.		Dist.		La	Long.		Dist.		La	Long.		Dist.	
Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P.	G.	P.	Gr	Gr.	P.	G.	P.
0		0		0	30	47	10	54	00	60	112	93	108	00
1	1	49	1	80	31	48	84	55	80	61	115	97	109	80
2	2	99	3	60	32	50	60	57	60	62	119	10	111	60
3	4	49	5	40	33	52	37	59	40	63	122	34	113	40
4	6	00	7	20	34	54	16	61	20	64	125	70	115	20
5	7	50	9	00	35	55	98	63	00	65	129	18	117	00
6	9	00	10	80	36	57	82	64	80	66	132	78	118	80
7	10	50	12	60	37	59	68	66	60	67	139	54	120	60
8	12	01	14	40	38	61	57	68	40	68	140	45	122	40
9	13	52	16	20	39	63	48	70	20	69	144	53	124	20
10	15	04	18	00	40	65	42	72	00	70	148	81	126	00
11	16	56	19	80	41	67	39	73	80	71	153	30	127	80
12	18	09	21	60	42	69	39	75	60	72	158	00	129	60
13	19	62	23	40	43	71	42	77	40	73	163	00	131	40
14	21	16	25	20	44	73	48	79	20	74	168	26	133	20
15	22	70	27	00	45	75	58	81	00	75	173	86	135	00
16	24	62	28	80	46	77	72	82	80	76	179	84	136	80
17	25	82	30	60	47	79	89	84	60	77	186	26	138	60
18	27	39	32	40	48	82	10	86	40	78	193	17	140	40
19	28	97	34	20	49	84	36	88	20	79	200	69	142	20
20	30	55	36	00	50	86	67	90	00	80	208	91	144	00
21	32	15	37	80	51	89	03	91	80	81	217	98	145	80
22	33	76	39	60	52	91	43	93	60	82	228	13	147	60
23	35	38	41	40	53	93	88	95	40	83	239	61	149	40
24	37	01	43	20	54	96	40	97	20	84	252	85	151	20
25	38	66	45	00	55	98	98	99	00	85	268	51	153	00
26	40	32	46	80	56	101	62	100	80	86	287	67	154	80
27	42	00	48	60	57	104	33	102	60	87	312	36	156	60
28	43	67	50	40	58	107	12	104	40	88	345	15	158	40
29	45	38	52	20	59	109	98	106	20	89	406	72	160	20
30	47	10	54	00	60	112	93	108	00	90				

The sixt Rumbe from the Meridian.					East North-east, West North-west.					East South-east, West South-west.				
La	Long.		Dist.		La	Long.		Dist.		La	Long		Dist.	
Gr	G.	P.	Gr.	P.	Gr	Gr.	P.	G.	P.	Gr	Gr.	P.	Gr.	P.
0		0		0	30	75	98	78	39	60	182	18	156	78
1	2	41	2	61	31	78	78	81	00	61	187	07	155	40
2	4	83	5	23	32	81	61	83	62	62	192	13	162	01
3	7	25	7	84	33	84	48	86	23	63	197	36	164	62
4	9	66	10	45	34	87	37	88	84	64	202	77	167	24
5	12	08	13	06	35	90	30	91	46	65	208	38	169	85
6	14	51	15	68	36	93	27	94	07	66	214	20	172	46
8	16	94	18	29	37	96	27	96	68	67	220	25	175	08
7	19	37	20	90	38	99	31	99	30	68	226	57	177	69
9	21	81	23	52	39	102	40	101	91	69	233	15	180	30
10	24	26	26	13	40	105	53	104	52	70	240	06	182	92
11	26	71	28	74	41	108	71	107	14	71	247	27	185	53
12	29	17	31	36	42	111	93	109	75	72	254	90	188	14
13	31	65	33	97	43	115	20	112	36	73	262	92	190	75
14	34	14	36	58	44	118	53	114	97	74	271	43	193	37
15	36	63	39	20	45	121	92	117	59	75	280	46	195	98
16	39	13	41	81	46	125	36	120	20	76	290	11	198	59
17	41	65	44	42	47	128	87	122	81	77	300	46	201	21
18	44	18	47	03	48	132	44	125	43	78	311	62	203	82
19	46	73	49	65	49	136	09	128	04	79	323	73	206	43
20	49	29	52	26	50	139	81	130	65	80	337	00	209	05
21	51	87	54	87	51	143	60	133	27	81	351	64	211	66
22	54	47	57	49	52	147	47	135	88	82	368	00	214	27
23	57	08	60	10	53	151	44	138	49	83	386	51	216	89
24	59	71	62	71	54	155	50	141	10	84	407	89	219	50
25	62	36	65	33	55	159	66	143	72	85	433	13	222	11
26	65	04	67	94	56	163	93	146	33	86	464	05	224	73
27	67	74	70	55	57	168	31	148	95	87	503	88	227	34
28	70	46	73	17	58	172	80	151	56	88	560	00	229	95
29	73	20	75	78	59	177	42	154	17	89	656	08	232	56
30	75	98	78	39	60	182	18	156	78	90				

The seventh Rumbe from the Meridian.					East and by North, West and by North,					East and by South, West and by South.				
La	Long.		Dist.		La	Long.		Dist.		La	Long.		Dist.	
Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P.	Gr.	P.
0		0		0	30	158	23	153	77	60	379	35	307	55
1	5	02	5	12	31	164	[illegible]	158	90	61	389	56	312	67
2	10	05	10	25	32	169	96	164	02	62	400	10	317	80
3	15	08	15	38	33	175	92	169	15	63	410	98	322	93
4	20	12	20	50	34	181	95	174	28	64	422	26	328	05
5	25	16	25	63	35	188	04	179	40	65	433	94	333	18
6	30	21	30	75	36	194	22	184	53	66	446	03	338	30
7	35	27	35	88	37	200	48	189	65	67	458	66	343	43
8	40	34	41	00	38	206	82	194	78	68	471	80	348	55
9	45	42	46	13	39	213	24	199	90	69	485	52	353	68
10	50	52	51	26	40	219	76	205	03	70	499	89	358	81
11	55	63	56	38	41	226	37	210	16	71	514	94	363	93
12	60	77	61	51	42	233	08	215	28	72	530	79	369	06
13	65	92	66	63	43	239	90	220	41	73	547	52	374	18
14	71	09	71	76	44	246	84	225	53	74	565	22	379	31
15	76	28	76	88	45	253	89	230	66	75	584	03	384	43
16	81	50	82	01	46	261	05	235	79	76	604	13	389	56
17	86	75	87	14	47	268	36	240	91	77	625	67	394	69
18	92	02	92	26	48	275	80	246	04	78	648	91	399	81
19	97	31	97	39	49	283	40	251	16	79	674	15	404	94
20	102	64	103	51	50	291	13	256	29	80	701	75	410	06
21	108	01	107	64	51	299	03	261	41	81	732	25	415	19
22	113	42	112	77	52	307	11	266	54	82	766	30	420	32
23	118	87	117	89	53	315	37	271	69	83	804	86	425	44
24	124	35	123	02	54	323	82	276	79	84	849	38	430	57
25	129	87	128	14	55	332	48	281	92	85	901	98	435	69
26	135	44	133	27	56	341	36	287	04	86	966	31	440	82
27	141	05	138	40	57	350	47	292	17	87	1049	26	445	94
28	146	71	143	52	58	359	81	297	30	88	1166	11	451	07
29	152	44	148	65	59	369	45	302	42	89	1366	23	456	20
30	158	23	153	77	60	379	35	307	51	90				

The eight Rumbe of East and West, with the Longitude answering to one degree of distance, and the distance belonging to one degree of Longitude.

La	Long.		Dist.		La	Long.		Dist.		La	Long.		Dist.	
Gr	Gr.	P.	Parts.		Gr	Gr.	P.	Parts.		Gr	Gr.	P.	Parts.	
0		0	100	00	30	1	25	86	60	60	2	00	50	00
1	1	00	99	98	31	1	17	85	71	61	2	06	48	48
2	1	00	99	94	32	1	18	84	80	62	2	13	46	94
3	1	00	99	86	33	1	19	83	86	63	2	20	45	40
4	1	00	99	75	34	1	21	82	90	64	2	28	43	83
5	1	00	99	62	35	1	22	81	91	65	2	37	42	26
6	1	01	99	45	36	1	24	80	90	66	2	46	40	67
7	1	01	99	25	37	1	25	79	86	67	2	56	39	07
8	1	01	99	02	38	1	27	78	80	68	2	67	37	46
9	1	01	98	76	39	1	29	77	71	69	2	79	35	83
10	1	02	98	48	40	1	31	76	60	70	2	92	34	20
11	1	02	98	16	41	1	33	75	47	71	3	07	32	55
12	1	02	97	81	42	1	35	74	31	72	3	24	30	90
13	1	03	97	43	43	1	37	73	13	73	3	42	29	23
14	1	03	97	03	44	1	39	71	93	74	3	63	27	56
15	1	03	96	59	45	1	41	70	71	75	3	86	25	88
16	1	04	96	12	46	1	44	69	46	76	4	13	24	19
17	1	04	95	63	47	1	47	68	20	77	4	44	22	49
18	1	05	95	10	48	1	49	66	91	78	4	81	20	79
19	1	06	94	55	49	1	52	65	60	79	5	24	19	08
20	1	06	93	97	50	1	55	64	28	80	5	76	17	36
21	1	07	93	35	51	1	59	62	93	81	6	39	15	64
22	1	08	92	72	52	1	62	61	56	82	7	18	13	91
23	1	09	92	05	53	1	66	60	18	83	8	20	12	18
24	1	09	91	35	54	1	70	58	77	84	9	57	10	45
25	1	10	90	63	55	1	74	57	35	85	11	47	8	71
26	1	11	89	88	56	1	79	55	92	86	14	33	6	97
27	1	12	89	10	57	1	84	54	46	87	19	11	5	23
28	1	13	88	29	58	1	89	52	99	88	28	65	3	49
29	1	14	87	46	59	1	94	51	50	89	57	30	1	74
30	1	15	86	60	60	2	00	50	00	90				0

These tables are calculated for each of the Rumbs. The first seuen haue three columnes, and of them the first containeth the degrees of Latitude, from the Equinoctiall to the Pole: the second doth giue the difference of Longitude; and the third the distance, both of them belonging to that Rumb and latitude.

As in the Table of the third Rumb; at the *latitude* of 50 *Gr.* I find vnder the title of *Longitude* 38 *Gr.* 69 *parts*, and vnder the title of *Distance* 60 *Gr.* 13 *parts*. This shewes that if the course held constantly on the third Rumb from the Equinoctiall to the Latitude of 50 *Gr.* the difference of Longitude would be 38 *Gr.* 69 *parts* of a 100, and the distance vpon the Rumbe 60 *Gr.* 13 *parts*. For here I reckon the distance by degrees, rather then by leagues or miles, and subdiuide each degree into 100 parts, rather then into 60 minutes, for the more ease in calculation, and withall to make the calculation to agree the better, both with this, and my *Crosse-staffe*, and other instruments.

The vse of these Tables, for the finding of the difference of Longitude, is this. Turne to the table of the *Rumb*, and there see what longitude belongeth to either latitude, then take the one longitude out of the other, the remainder will be the difference of longitude required.

As in the former example, where the places giuen were *A*, in the latitude of 50 *Gr.* *C* in the latitude of 55 *Gr.* and the *Rumb* the third from the meridian: I looke into the table of the third *Rumb* and there find,

Latitude 50 *gr.*	Longitude	38 *gr.*	69 *parts.*
Latitude 55.	Longitude	44.	19.
Therefore the diff. of Longitude		5	50

There is another vse of these tables, for the describing of the *Rumbs* both on the *Globe*, aud all sorts of *Charts*. For hauing drawne the circles of longitude and latitude, and finding by the tables, the difference of longitude belonging to each *Rumb* and latitude: If we make a prick in the chart, at

eucry

euery degree of latitude, according to that difference of longitude, and draw lines through those prickes, so as they make no angles, the lines so drawne shall be the *Rumbs* required.

The vse of the eight *Rumb* is something different from the rest. For there being here no change of latitude, I haue set to each latitude, the difference of longitude, belonging to one degree of distance, and the distance belonging to one degree of longitude.

As if two places shall be 20 leagues, or one degree distant one from the other, in the latitude of 50 *gr.* the difference of longitude betweene them will be 1 *gr.* 55 *parts*. But if they differ one degree in longitude, the distance betweene them will be onely 64 parts, which fall short of 13 leagues, or at the most 6428 parts, such as 10000 do make a degree.

6 *By the difference of longitude, Rumb, and one latitude, to find the other latitude.*

As if the places giuen were *A*, in the latitude of 50 *gr.* *C* in a greater latitude but vnknowne, the difference of longitude 5 *gr.* $\frac{1}{2}$, and the Rumb the third from the Meridian.

In the chart let *A B*, DC, meridians, be drawne through *A* and *C*, according to the difference of longitude, one 5 *gr.* $\frac{1}{2}$ from the other, and a parallell of latitude through *A*, crossing the meridian C D in D: then in *A*, with *A B*, make an angle of the Rumb *B A C*: so the degrees in the meridian betweene D and *C*, shall be found to be 5 *gr.* the proper difference of latitude which was required. Wherefore the proportion holds for the *Sector*,

As A D the Radius,
to DC the tangent of the Rumb from the equator:
So A D as difference of longitude,
to D C the proper difference of latitude.

According to this, I take 56 *gr.* 15 *m.* for the angle of the Rumb from the equator, out of the greater *Tangent*, and make

make it a parallell Radius. Then I reckon 5 *gr.*½ in the line of *lines* from the center, for the difference of longitude. So the parallell taken from the termes of this difference, and measured in the line of *meridians*, shal reach from 50 *gr.* the latitude giuen, to 55 *gr.* which is the latitude required.

Or if the Rumb fall nearer to the meridian.

As *BC* the tangent of the Rumb from the meridian,
is to *A B* the Radius:
So *B C* as difference of longitude,
to *A* D the proper difference of latitude.

According to this we may best work by parallel entrance; first take 33 *gr.* 45 *m.* for the angle of the Rumb from the meridian, out of the greater *Tangent*, and make it a parallell Radius; then take 5 *gr.*½ for the difference of longitude out of the line of *lines*, and carrie it parallell to the former, till the feete of the compasses stay in like points: so the line between the center and the place of this stay, being taken and measured in the line of *meridians* from 50 *gr.* forward, shall shew the latitude required to be 55 *gr.* as in the former way.

The like may be found by the tables of Rumbs. For in the table of the third Rumb, at the latitude of 50 *gr.* I finde the longitude of 38 *gr.* 69 *p*; to this if I adde 5 *gr.* 50 *p.* for the difference of longitude giuen, the compound longitude will be 44 *gr.* 19 *p.* and this answers to the latitude of 55 *gr.*

But if this difference of latitude were to be found by the common sea-chart, it should seeme to be 8 *gr.* 13 *m*; and so the second latitude should be 58 *gr.* 13 *m.* which is aboue 3 *gr.* more then the truth.

7 *By one latitude, rumb, and distance, to find the difference of longitude.*

As if the places giuen were *A* in the latitude of 50 *gr.* *C* in a greater latitude but vnknowne, the distance vpon the Rumb being 6 *gr.* betweene them, and the Rumb the third from the meridian.

In

In the chart, let a meridian *A B*, and a parallell A D be drawne through *A*; and in *A*, with A B, make an angle BAC for the Rumb from the meridian; then open the compasses according to the latitude of the places to *E F*, the quantitie of *6 gr.* in the meridian, transferring them into the Rumb from *A* to *C*, and through *C* draw another meridian D C, crossing the parallell drawne through *A* in D: so the degrees intercepted in the parallell from *A* to D, shall shew the difference of longitude required to be about 5 *gr.* $\frac{1}{2}$. Wherefore the proportion holds for the Sector.

As *A C* the Radius,
is to AD, equall to B C, the sine of the Rumb from the (meridian:
So *A C* as proper distance vpon the *R*umb,
to A D the difference of longitude.

According to this I take the sine of 33 *gr.* 45 *m.* for the angle of the *R*umb from the meridian, and make it a parallell *R*adius; then keeping the *Sector* at this angle, I take *6 gr.* for the distance out of the *meridian* line, according to the estimated latitudes of both places, and lay it on both sides of the *Sector* from the center: so the parallell taken from the termes of this distance, and measured in the lines of *lines*, shall shew the difference of longitude to be about 5. *gr.* $\frac{1}{2}$.

In this, and some of the *Prop.* following, where there is but one latitude knowne, there may be sometimes an error of a minute or two, in the estimation of the proper distance, yet it may be rectified at a second operation.

This proposition may also be wrought by the Tables of *R*umbs. For according to the example, in the Table of the third *R*umb, at the latitude of 50 *gr.* I find the longitude of 38 *gr.* 69 *p.* and the distance of 60 *gr.* 13 *p.* to this I adde 6 *gr.* for the distance giuen; so the compound distance will be 66 *gr.* 13 *p.* and this answers to the longitude of 44 *gr.* 19 *p*; then if I take the one longitude out of the other, the difference will be 5 *gr.* 50 *p.* as before.

But if this difference were to be found by the common sea-chart, it should seeme to be onely 3 *gr.* 20 *m.* which is

more then 2 *gr.* lesse then the truth.

8 *By one latitude, Rumb, and difference of longitudes, to find the distance.*

As if the places giuen were *A*, in the latitude of 50 *gr.* C in a greater latitude but vnknowne, the difference of longitude betweene them being 5 *gr.* $\frac{1}{2}$, and the Rumb the third from the meridian.

In the chart let A B, D C, meridians be drawne through *A* and *C*, according to the difference of longitude, and a parallell of latitude through *A*, crossing the meridian DC in D; then in *A*, with A B, make an angle of the Rumb BAC: so the distance on the Rumb from *A* to *C* taken and measured in the *meridian*, according to the estimated latitude of the places, shall be found to be 6 *gr.* Wherefore the proportion holds for the *Sector.*

As A D, equall to BC, the sine of the Rumb from the meri-(dian,
is to *A C* the Radius:
So A D as difference of longitudes,
to *A C* the proper distance vpon the Rumb.

According to this, I take the laterall Radius, and make it a parallell sine of 33 *gr.* 45 *m.* which is here the angle of the Rumb from the meridian; then I reckon 5 *gr.* $\frac{1}{2}$ in the lines of *lines* from the center, for the difference of longitude: so the parallell taken from the termes of this difference, and measured in the line of *meridians*, according to the latitudes of the places, shall there shew the distance required to be about 6 *gr.* which are 120 leagues.

Or if the Rumb fall nearer to the meridian, that the lateral Radius cannot be fitted ouer in his sine, this *Prop.* must be wrought by parallell entrance, and so also it giues the same distance as before.

Or we may find this distance by the Table of Rumbs. For in the table of the third Rumb, at the latitude of 50 *gr.* I find the longitude of 38 *gr.* 69 *p.* and the distance of 60 *gr.* 13 *p.*

To

To this longitude here found, I adde 5 *gr.* 50 *p.* for the difference of longitude giuen: so the compound longitude will be 44 *gr.* 19 *p.* and this answers to the distance of 66 *gr.* 15 *p.* Then if I take the one distance out of the other, the remainder will be 6 *gr.* 02 *p.* for the distance required.

But if this distance were to be measured on the common sea-chart, it should seeme to be almost 10 *gr.* or at the least 197 leagues, aboue 77 leagues more then the truth.

9 *By one latitude, distance, and difference of longitudes, to find the Rumb.*

As if the places giuen were *A*, in the latitude of 50 *gr.* *C* in a greater latitude but vnknowne, the difference of longitude betweene them being 5 *gr.* $\frac{1}{2}$, and the distance 6 *gr.* vpon the *Rumb.*

In the chart let AB, DC, meridians, be drawne through *A* and *C*, and a parallell of latitude through *A*; then open the compasses according to the latitudes of the places, to *E F* the quantitie of 6 *gr.* in the *meridian*, and setting the one foote in *A*, the other foote shall crosse the other meridian in C; and if we draw the right line *A C*, the angle BAC shall shew the inclination of the *Rumb* to the meridian to be about 33 *gr.* 45 *m.* Wherefore the proportion holds for the *Sector.*

As *A C* the proper distance vpon the *Rumb*,
is to A D the difference of longitude:
So *A C* as *Radius*,
to AD, equall to B C, the sine of the *Rumb* from the meridian.

According to this, I take the proper distance 6 *gr.* out of the line of *meridians*, and lay it on both sides of the *Sector* from the center; then I take the difference of longitude 5 *gr.* $\frac{1}{2}$ out of the line of *lines*, and to it open the *Sector* in the terms of the former distance: so the parallell *Radius* taken from betweene 90 and 90, and measured in the *sines*, doth giue about 33 *gr.* 45 *m.* for the *Rumb* required.

But if this *Rumb* were to be found by the common sea-

chart, it should seeme to be aboue *66 gr.* and so almost the sixt *R*umb from the meridian.

10 *By the longitude and latitude of two places, to find their distance vpon the Rumb.*

Let the *Sector* be opened in the lines of *lines*, vnto a right angle (as was shewed before *Cap. 2. Prop. 7*;) then take out the proper difference of latitude, and lay it on the one line, and the difference of longitude, and lay it on the other line, so as they may both meete in the center, marking how far they extend. For the line taken from the termes of their extension, and measured in the *meridian*, according to their latitudes, shall shew the distance required.

So if the places giuen were *A* and *C*, *A* in the latitude of *50 gr. C* in the latitude of *55 gr.* the proper difference of latitude shall be the line *A B*, and let *B C* the difference of longitude be *5 gr.* $\frac{1}{2}$, we shall find that *A C* the distance vpon the Rumb is about *6 gr.* which make 120 leagues.

For in the chart, let an occult meridian be drawne through *A*, and a parallell of latitude through *C*, crossing the former meridian in *B*, and a right line for the Rumb from *A* to *C*, so haue we a rectangle triangle *A B C*, whose base *A C*, taken and measured in the meridian from *E* below *50 gr.* to *F*, as much aboue *55 gr.* doth containe the quantitie of *6 gr.*

In the same maner the *Sector* being opened to a right angle, in the lines of *lines*: if we take the difference of latitude out of the line of *meridians*, in his proper place from *50 gr.* to *55 gr.* and place it on one of the sides from the center, to resemble *A B*, then reckon the difference of longitude on the other perpendicular line from the center to *5 gr.* $\frac{1}{2}$, in stead of *B C*, we shall haue the like rectangle triangle on the *Sector*, to that which we had before on the chart; and if we take out the base of it, and measure it in the line of *meridians* from below *50 gr.* to as much aboue *55 gr.* we shall finde as before, that it containeth about *6 gr.* or 120 leagues.

But if this distance were to be measured on the common sea-

sea-chart, it should seeme to be almost 7 *gr.* $\frac{1}{4}$, or 145 leagues; which is 25 leagues more then the truth.

11 *By the latitude of two places, and the distance vpon the Rumb, to find the difference of longitude.*

Let the *Sector* be opened in the lines of *lines* to a right angle, then take out the proper difference of latitudes, and lay it on one of the lines from the center, then take the proper distance with a paire of compasses, and setting one foote in the termes of the difference, turne the other foote to the other line of the *Sector*, and it shall there shew the difference of longitude required.

So if the places giuen were *A*, in the latitude of 50 *gr.* *C* in the latitude of 55 *gr.* with 6 *gr.* of distance one from another, we shall find their difference of longitude to be about 5 *gr.* $\frac{1}{2}$.

For in the chart let a meridian *A B* be drawne for the one, and *B C*, *A D*, parallels of latitude for them both. Then open the compasses according to the latitude of the places, to *E F* the quantitie of 6 *gr.* in the *meridian*, and setting one foote in *A*, hauing latitude of 50 *gr.* turne the other to the parallell of 55 *gr.* and it shall there cut off the required difference of longitude *B C* 5 *gr.* $\frac{1}{2}$.

In the same maner, the *Sector* being opened to a right angle, in the lines of *lines*: if we take the difference of latitude out of the line of *meridians* in his proper place from 50 *gr.* vnto 55 *gr.* and place it on one of the lines from the center; then take 6 *gr.* the distance vpon the Rumb out of the same line of *meridians*, according to the latitudes of the places, and set the one foote in the terme of the former difference, turning the other foote to the other perpendicular line, we shall finde that it will crosse it about 5 *gr.* $\frac{1}{2}$ from the center: which is the difference of longitude required.

But if this difference of longitude were to be found by the common sea-chart, it would seeme to be only 3 *gr.* 20 *m.* which is more then 2 *gr.* 10 *m.* lesse then the truth.

12 *By one latitude, distance and difference of longitudes, to finde the difference of latitudes.*

Let the *Sector* be opened in the line of *lines* to a right angle, and let the difference of longitude be reckoned in one of those lines from the center; then take the proper distance with a paire of compasses, and setting the one foote in the terme of the former difference, turne the other foote to the other line of the *Sector*, and it shal thence cut off a line, equal to the proper difference of latitude required.

So if the places giuen were *A* and *C*, *A* in the latitude of 50 *gr.* *C* in a greater latitude but vnknowne, the difference of longitude betweene them 5 *gr.* $\frac{1}{3}$, and the distance vpon the Rumb 6 *gr.* or 120 leagues, we shall find the difference of latitude to be 5 *gr.*

For in the chart, let occult meridians be drawne through *A* and *C*, and a parallell of latitude through *A*; then open the compasses according to the estimated latitudes of the places to *E F* the quantitie of 6 *gr.* in the meridian, and setting the one foote in *A*, turne the other to the meridian drawne through *C*, and it shall there cut off the line *D C*, which is the difference of latitude required.

In the same maner, the *Sector* being opened to a right angle, in the lines of *lines*, if in the one line we reckon the difference of longitude from the center to 5 *gr.* $\frac{1}{3}$, then taking 6 *gr.* for the distance out of the line of *Meridians*, according to the latitude of the places, we set the one foote in the terme of the giuen difference, and turne the other foote to the other perpendicular line, we shall finde that it cuts a line from it, which taken and measured in the line of *meridians*, from 50 *gr.* on forward, doth shew the difference of latitude to be as before 5 *gr.*

But if this difference of latitude were to be found by the common sea-chart, it would seeme to be only 2 *gr.* 25 *m.* which is 2 *gr.* 35 *m.* lesse then the truth. Such is the difference betweene both these charts.

THE

THE THIRD BOOKE

Containing the vſe of the particular *Lines.*

THE lines of *lines*, of *ſuperficies*, of *ſolids*, of *ſines*, with the laterall lines of *tangents* and *meridians*, whereof I haue hitherunto ſpoken, are thoſe which I principally intended: that little roome on the *Sector* which remaineth, may be filled vp with ſuch particular lines as each one ſhall think conuenient for his purpoſe. I haue made choiſe of ſuch as I thought might be beſt prickt on without hindring the ſight of the former, viz. lines of *Quadrature*, of *Segments*, of *Inſcribed bodies*, of *Equated bodies*, and of *Mettals*.

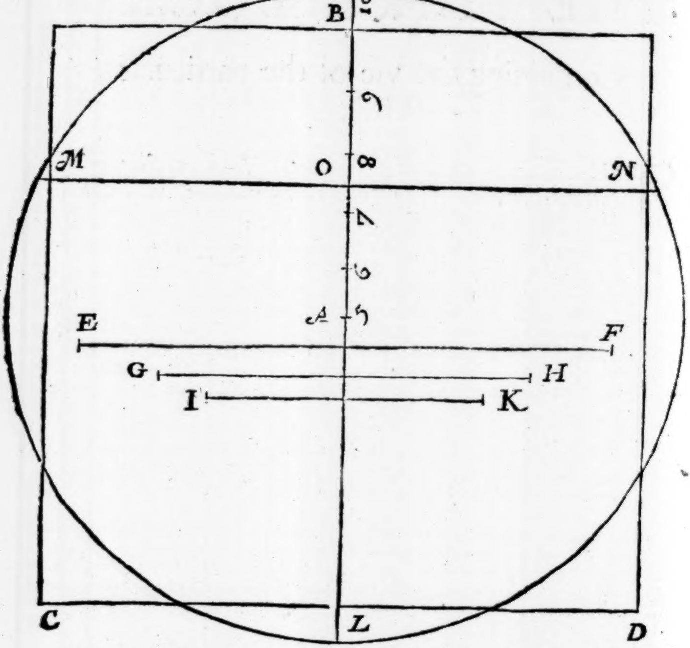
CHAP. I.

Of the lines of Quadrature.

THe lines of *quadrature* may be knowne by the letter *Q*, and by their place betweene the lines of *ſines*. *Q* ſignifieth the ſide of a *ſquare*; 5 the ſide of a *pentagon* with fiue equall ſides, 6 of an *hexagon* with ſix equall ſides, and ſo 7, 8, 9, and 10. *S* ſtands for the Semidiameter of a circle, and 90 for a line equall to 90 *gr.* in the circumference. The vſe of them may be

1 *To make a ſquare equall to a circle giuen.*
2 *To make a circle equall to a ſquare giuen.*

If the circle be firſt giuen, take his ſemidiameter, and to it open the *Sector* in the points at *S*: ſo the parallell taken from betweene the points at *Q*, ſhall be the ſide of the ſquare required.



If the square be giuen take his side, and to it open the *Sector*, in the points at *Q*: so the parallell taken from betweene the points at *S*, shall be the Semidiameter of the circle required.

Let the Semidiameter of the circle giuen be *A B*, the side of the square equall vnto it shall be found to be *C D*.

3 *To reduce a circle giuen, or a square into an equall pentagon, or other like sided and like angled figure.*

Take the side of the figure giuen, and fit it ouer in his due points: so the parallels taken from betweene the points of the

the other figures, ſhall be the ſides of thoſe figures: which being made vp with equall angles, ſhall be all equall one to the other.

Let the Semidiameter of the circle giuen be *AB*, the ſide of an *hexagon* equal to this circle, ſhall by theſe meanes be found to be *GH*; and the ſides of an octagon to be *IK*. Other planes not here ſet downe, may firſt be reduced into a ſquare, by the ſixt *Prop. Superf.* and then into a circle, or other of theſe equall figures, as before.

4 *To find a right line, equall to the circumference of a circle, or other part thereof.*

Take the Semidiameter of the circle giuen, and to it open the *Sector* in the points at *S*; ſo the parallell taken from betweene the points at 90 in this line, ſhall be the fourth part of the circumference: which being knowne, the other parts may be found out by the ſecond and third *Prop.* of *lines.*

Thus if the Semidiameter of the circle giuen be *A B*, the right line *E F* ſhall be found to be the fourth part of the circumference. Therefore the double of *E F* ſhall be equall to the circumference of 180 *gr*; and the halfe of *E F* ſhal be the circumference of 45 *gr.* and ſo in the reſt.

CHAP. II.

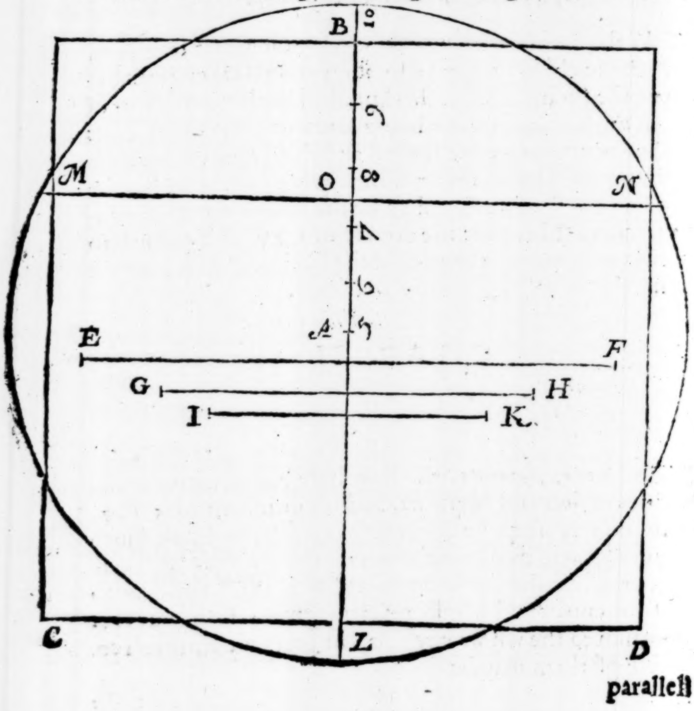
Of the lines of Segments.

THe lines of *ſegments* which are here placed betweene the lines of *ſines* and *ſuperficies*, and are numbred by 5, 6, 7, 8, 9, 10, do repreſent the diameter of a circle, ſo diuided into a hundred parts, as that a right line drawne through theſe parts, perpendicular to the diameter, ſhall cut the circle into two ſegments, of which the greater ſegment ſhall haue that proportion to the whole circle, as the parts cut haue to 100. The vſe of them may be

1 *To diuide a circle giuen into two ſegments, according to a proportion giuen.*
2 *To finde a proportion betweene a circle and his ſegments giuen.*

Let the *Sector* be opened in the points of an 100, to the diameter of the circle giuen: ſo a parallell taken from the points proportionall to the greater ſegment required, ſhall giue the depth of that greater ſegment.

Or if the ſegments be giuen, let the *Sector* be opened as before; then take the depth of the greater ſegment, and carry it



parallell

parallell to the diameter: so the number of points wherein they stay, shall shew the proportion to 100.

As if the diameter of the circle giuen were *B L*, the depth of the greater segment *L O* being 75, doth shew the proportion of the segment *O M L N* to the circle to be as 75 to 100. viz. three parts of foure.

Hence I might shew, if there were any vse of it,

To find the side of a square, equall to any knowne segment of a circle.

The side of a square equall to the whole circle, may be found by the former *Cap.* and then hauing the proportion of the segment to the circle, we may diminish the square in such proportion, by that which hath been shewed *Lib.* 1. *Cap.* 3. *Prop.* 3.

CHAP. III.

Of the lines of Inscribed bodies.

THe lines of *inscribed bodies* are here placed betweene the lines of *lines*, and may be knowne by the letters, *D, S, I, C, O, T*; of which *D* signifieth the side of a *dodecahedron*, *I* of an *Icosahedron*, *C* of a *cube*, *O* of an *octahedron*, and *T* of a *tetrahedron*, all inscribed into the same sphere, whose semidiameter is here signified by the letter *S*.

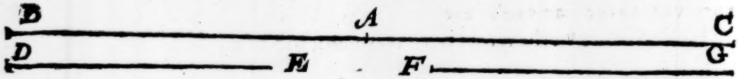
The vse of these lines may be,

1 *The semidiameter of a sphere being giuen, to find the sides of the fiue regular bodies, which may be inscribed in the said sphere.*

2 *The side of any of the fiue regular bodies being giuen, to find the semidiameter of a sphere, that will circumscribe the said bodie.*

If the sphere be first giuen, take his semidiameter, and to it

open the *Sector* in the points at *S*: if any of the other bodies be first giuen, take the side of it, and fit it ouer in his due points: so the parallell taken from betweene the points of the other bodies, shall be the sides of those bodies, and may be inscribed into the same sphere.



So if the semidiameter of the sphere be *A C*, the side of the *dodecahedron* inscribed shall be *D E*.

CHAP. IIII.

Of the lines of Equated bodies.

THe lines of *equated bodies* are here placed betweene the lines of *lines* and *solids*, noted with these letters, *D*, *I*, *C*, *S*, *O*, *T*, of which *D* stands for the side of a *dodecahedron*, *I* for the side of an *Icosahedron*, *C* for the side of a *cube*, *S* for the diameter of a *sphere*, *O* for the side of an *octahedron*, and *T* for the side of a *tetrahedron*, all equall one to the other. The vse of these lines may be

1 *The diameter of a sphere being giuen, to find the sides of the fiue regular bodies, equall to that sphere.*

2 *The side of any of the fiue regular bodies being giuen, to find the diameter of a sphere, and the sides of the other bodies, equall to the first body giuen.*

If the sphere be first giuen, take his diameter, and to it open the *Sector* in the points at *S*: if any of the other bodies be first giuen, take the side of it, and fit it ouer in his due points: so the parallels taken from between the points of the other bodies, shall be the sides of those bodies equall to the first body giuen.

Thus in the last diagram, if the diameter of a sphere giuen be *B C*, the side of the *dodecahedron* equall to this sphere, would be found to be *FG*.

CHAP. V.

Of the Lines of Mettalls.

THe lines of *Mettalls* are here ioyned with those before of *equated bodies*, and are noted with these characters ☉. ☿. ♄. ☽. ♀. ♂. ♃. of which ☉ stands for gold, ☿ for quicksiluer, ♄ for leade, ☽ for siluer, ♀ for copper, ♂ for iron, and ♃ for tin. The vse of them is to giue a proportion betweene these seuerall mettalls, in their magnitude and weight, according to the experiments of *Marinus Ghetaldus*, in his booke called *Promotus Archimedes*.

1 *In like bodies of seuerall mettalls and equall weight, hauing the magnitude of the one, to finde the magnitude of the rest.*

Take the magnitude giuen out of the lines of *Solids*, and to it open the *Sector* in the points belonging to the mettall giuen: so the parallells taken from betweene the points of the other mettalls, and measured in the lines of *Solids*, shall giue the magnitude of their bodies.

Thus hauing cubes or spheres of equall weight, but seuerall mettalls, we shall finde that if those of tin containe 10000 *D*, the others of iron wil contain 9250, those of copper 8222, those of siluer 7161, those of lead 6435, those full of quicksiluer 5453, and those of gold 3895.

2 *In like bodies of seuerall mettalls and equall magnitude, hauing the weight of one to finde the weight of the rest.*

This proposition is the conuerse of the former, the proportion not direct, but reciprocall, wherefore hauing two like bodies, take the giuen weight of the one out of the lines of *Solids*, and to it open the *Sector* in the points belonging to

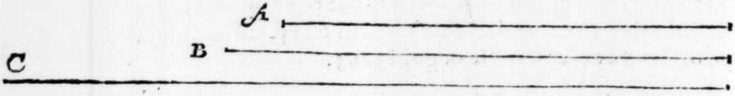
the mettall of the other body: so the parallell taken from the points belonging to the body giuen, and measured in the lines of *Solids*, shall giue the weight of the body required.

As if a cube of gold weighed 38 ℔. and it were required to know the weight of a cube of lead hauing equal magnitude. First I take 38 ℔. for the weight of the golden cube, out of the lines of *Solids*, & put it ouer in the points of ♄ belonging to lead: so the parallell taken from betweene the points of ☉ standing for gold, and measured in the lines of *Solids*, doth giue the weight of the leaden cube required to be 23 ℔.

Thus if a sphere of gold shall weigh 10000, we shall finde that a sphere of the same diameter full of quicksiluer shall weigh 7143, a sphere of lead 6053, a sphere of siluer 5438, a sphere of copper 4737, a sphere of iron 4210, and a sphere of tin 3895.

3 *A bodie being giuen of one mettall, to make another like vnto it, of another mettall, and equall weight.*

Take out one of the sides of the bodie giuen, and put it ouer in the points belonging to his mettall: so the parallell taken from between the points belonging to the other mettall, shall giue the like side, for the bodie required. If it be an irregular bodie, let the other like sides be found out in the same manner.



Let the bodie giuen be a sphere of lead containing in magnitude 16 D, whose diameter is *A*, to which I am to make a sphere of iron, of equall waight: If I take out the diameter *A*, and put it ouer in the points of ♄ belonging to lead, the parallell taken from betweene the points of ♂ standing for iron, shall be *B*, the diameter of the iron sphere required. And this compared with the other diameter, in the lines of *Solids*,

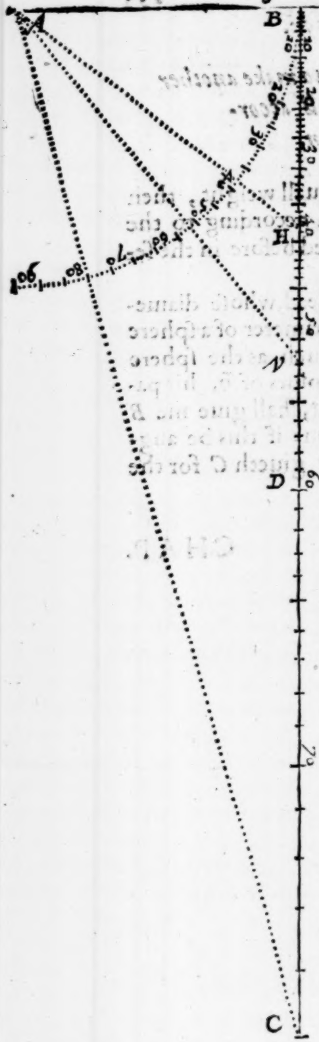
solids will be found to be 23 *d.* in magnitude.

4 *A body being giuen of one mettall, to make another like vnto it of another mettall, according to a weight giuen.*

First find the sides of a like bodie of equall weight, then may we either augment or diminish them according to the proportion giuen by that which we shewed before in the second and third *Prop.* of *Solids.*

As if the bodie giuen were a sphere of lead, whose diameter is *A*, and it were required to find the diameter of a sphere of iron, which shall weigh three times as much as the sphere of lead: I take *A*, and put it ouer in the points of ♄, his parallell taken from betweene the points of ♂, shall giue me *B* for the diameter of an equall sphere of iron: if this be augmented in such proportion as 1 vnto 3, it giueth *C* for the diameter required.

CHAP.



CHAP. VI.

Of the lines on the edges of the Sector.

HAuing shewed some vse of the lines on the flat sides of the *Sector*, there remaine onely those on the edges. And here one halfe of the outward edge is diuided into inches, and numbred according to their distance from the ends of the *Sector*. As in the Sector of fourteene inches long, where we find 1 and 13, it sheweth that diuision to be 1 inch from the nearer end, and 13 inches from the farther end of the *Sector*.

The other halfe containeth a line of lesser *tangents*, to which the gnomon is Radius. They are here continued to 75 *gr*. And if there be need to produce them farther, take 45 out of the number of degrees required, and double the remainder: so the *tangent* and *secant* of this double remainder being added, shall make vp the *tangent* of the degrees required.

As if *A B* being the Radius, and *B C* the tangent line, it were required to find the tangent of 75 *gr*. If we take 45 *gr*. out of 75 *gr*. the remainer is 30 *gr*. and the double 60 *gr*. whose tangent is B D, and the secant is A D: if then we adde A D to B D, it maketh *B C* the tangent of 75 *gr*. which was required. In like sort the secant of 61 *gr*. added to the tangent of 61 *gr*. giueth the tangent of 75 *gr*. 30 *m*. and the secant of 62 *gr*. added to the tangent of 62 *gr*. giueth the tangent of 76 *gr*.

and

and so in the rest. The vse of this line may be

To obserue the altitude of the Sunne.

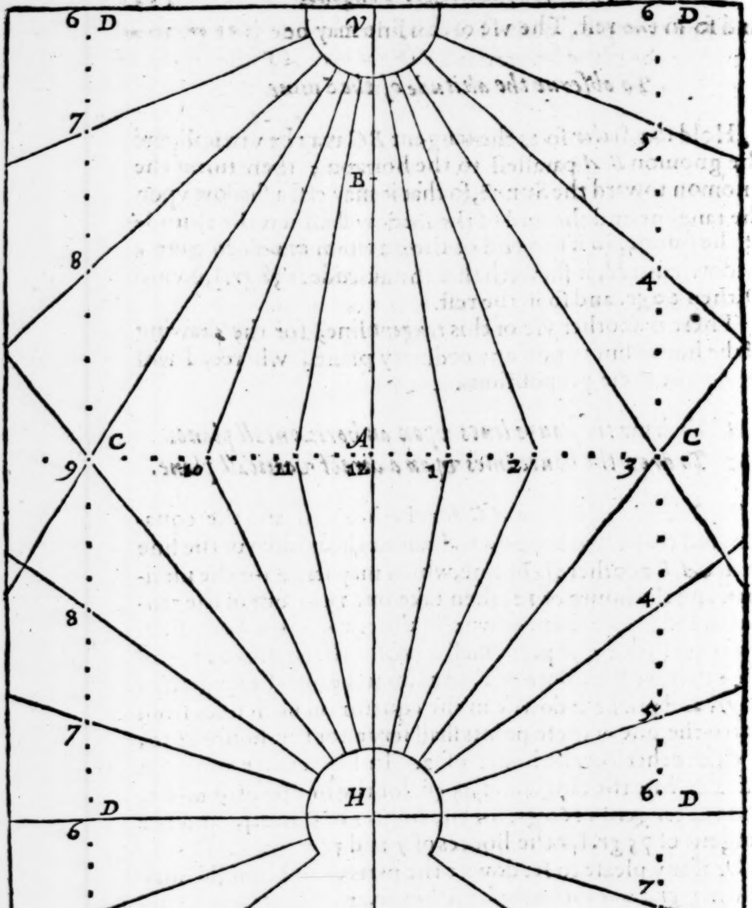
Hold the *Sector* so as the tangent *BC* may be verticall, and the gnomon *B A* parallell to the horizon; then turne the gnomon toward the Sunne, so that it may cast a shadow vpon the tangent, and the end of the shadow shal shew the altitude of the Sunne. So if the end of the gnomon at *A*, do giue a shadow vnto *H*, it sheweth that the altitude is 38 *gr.* $\frac{1}{3}$, if vnto D, then 60 *gr.* and so in the rest.

There is another vse of this *tangent* line, for the drawing of the houre lines vpon any ordinary plane, whereof I will set downe these propositions.

1 *To draw the houre lines vpon an horizontall plane.*
2 *To draw the houre lines vpon a direct verticall plane.*

First draw a right line *A C* for the horizon and the equator, and crosse it at the point *A* about the middle of the line with *A B* another right line, which may serue for the meridian and the houre of 12; then take out 15 *gr.* out of the tangents, and pricke them downe in the equator on both sides from 12: so the one point shall serue for the houre of 11, and the other for the houre of 1. Againe, take out the tangent of 30 *gr.* and pricke it downe in the equator on both sides from 12: so the one of these points shall serue for the houre of 10, and the other for the houre of 2. In like maner may you prick downe the tangent of 45 *gr.* for the houres of 9 and 3, and the tangent of 60 *gr.* for the houres of 8 and 4, and the tangent of 75 *gr.* for the houres of 7 and 5.

Or if any please to set downe the parts of an houre, he may allow 7 *gr.* 30 *m.* for euery halfe houre, and 3 *gr.* 45 *m.* for euery quarter. This done, you are to consider the latitude of the place, and the qualitie of the plane: For the *secant* of the latitude shal be the semidiameter in a vertical plane, & the *secant* of the complement of the latitude in an horizontall plane.


6 D
V
6 D
7
5
B
8
4
C
A
C
9
10
11
12
1
2
3
8
4
7
5
H
6
D
D
6
7
5
8
4

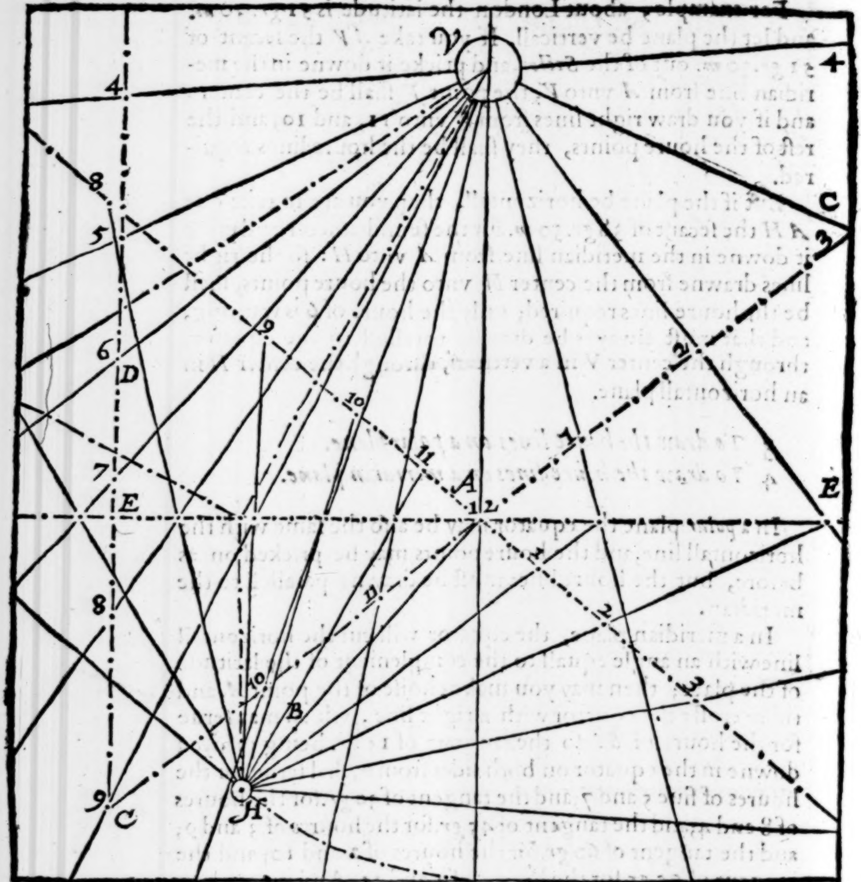
For example, about London the latitude is *51 gr.* 30 *m.* and let the plane be verticall. If you take *A V* the secant of *51 gr.* 30 *m.* out of the *Sector*, and pricke it downe in the meridian line from *A* vnto *V*, the point *V* shall be the center: and if you draw right lines from *V* vnto 11, and 10, and the rest of the houre points, they shall be the houre lines required.

But if the plane be horizontall, then you are to take out *A H* the secant of 38 *gr.* 30 *m.* for the semidiameter, and prick it downe in the meridian line from *A* vnto *H*: so the right lines drawne from the center *H* vnto the houre points, shall be the houre lines required; only the houre of 6 is wanting, and that must alwayes be drawne parallell to the equator, through the center V in a verticall, through the center *H* in an horizontall plane.

3 *To draw the houre lines on a polar plane.*
4 *To draw the houre lines on a meridian plane.*

In a *polar* plane the equator may be also the same with the horizontall line, and the houre points may be pricked on as before, but the houre lines must be drawne parallell to the meridian.

In a meridian plane, the equator will cut the horizontall line with an angle equall to the complement of the latitude of the place; then may you make choise of the point *A*, and there crosse the equator with a right line, which may serue for the houre of *6*: so the tangent of 15 *gr.* being pricked downe in the equator on both sides from 6, shal serue for the houres of fiue 5 and 7; and the tangent of 30 *gr.* for the houres of 8 and 4; and the tangent of 45 *gr.* for the houres of 3 and 9; and the tangent of 60 *gr.* for the houres of 2 and 10; and the tangent of 75 *gr.* for the houres of 1 and 11. And if you draw right lines through these houre points, crossing the equator at right angles, they shall be the houre lines required.



5 *To draw the houre lines in a verticall declining place.*

First, draw *AV* the meridian, and *AE* the horizontal line, crossing one the other at right angles in the point *A*.

2 Then

2 Then take out *A V*, the secant of the latitude of the place, which you may suppose to be 51 *gr.* 30 *m.* and prick it downe in the meridian line from *A* vnto *V*.

3 Because it is a declining plane, and you may suppose it to decline 40 *gr.* Eastward, you are to make an angle of the declination vpon the center *A*, below the horizontall line, and to the left hand of the meridian line, because the declination is Eastward, for otherwise it should haue bin to the right hand, if the declination had bin Westward.

4 Take *A H*, the secant of the complement of the latitude out of the *Sector*, & pricke it downe in the line of declination from *A* vnto *H*, as you did before for the semidiameter in the horizontall plane.

5 Draw a line at full length through the point *A*, which must be perpendicular vnto *A H*, and cut the horizontall line according to the angles of declination, and it will be as the equator in the horizontall plane.

6 Take the houre points out of the *Tangent* line in the *Sector*, and pricke them downe in this equator on both sides from the houre of 12 at *A*.

7 Lay your ruler, & draw right lines through the center *H*, & each of these houre points: so haue you all the houre lines of an horizontall plane, onely the houre of 6 is wanting, and that may be drawne through *H* perpendicular to *H A*.

Lastly you are to obserue and marke the intersections, which these houre lines do make with *A E* the horizontall line of the plane: and then if you draw right lines through the center *V*, and each of these intersections, they shal be the houre lines required.

6 *To pricke downe the houre points another way.*

Hauing drawne a right line for the equator as before, and made choice of the point *A*, for the houre of 12: you may at pleasure cut of two equal lines *A* 10, and *A* 2. Then vpon the distance betweene 10 and 2, make an equilaterall triangle, and you shall haue *B* for the center of your equator, and the

line *A B* shall giue the distance from *A* to 9, and from A to 3. That done take out the distance betweene 9 and 3, and this shall giue the distance from *B* vnto 8, and from 8 vnto 7, and from 8 vnto 1: and againe from B vnto 4, and from 4 vnto 5, and from 4 vnto 11. So haue you the houre points, and if you take out the distance B 1, B 3, B 5, &c. You may finde the points not onely for the halfe houres, but also for the quarters.

But if it so fall out, that some of these houre points fall out of your plane, you may helpe your selfe by the larger *tangent*, both in the verticall, and horizontall planes.

For if at the houre points of 3 and 9, you draw occult lines parallell to the meridian; the distances D C, betweene the houre line of 6, and the houre points of 3 and 9, will be equal to the semidiameter A V in a verticall, and AH in a horizontall plane, and if they be diuided in such sort as the line A C is diuided, you shall haue the points of 4, and 5, and 7, and 8, with their halfes and quarters.

As in the horizontall plane, take out the semidiameter A H, and make it a parallell Radius by fitting it ouer in the *sines* of 90 and 90: Then take 15 *gr.* out of the larger *tangent*, and lay them on the lines of *sines*, where they will reach from the center vnto the sines of 15 *gr.* 32 *m.* therefore take out the parallell sine of 15 *gr.* 32 *m.* and it shall giue the distance from 6 vnto 5, and from 6 vnto 7, in your horizontall plane. That done take out 30 *gr.* out of the larger *tangent*, and lay them on the *sines*, from the center vnto the sines of 35 *gr.* 16 *m.* and the parallell sine of 35 *g.* 16 *m.* shall giue you the distance from 6 vnto 4, and from 6 vnto 8, in your horizontall plane. The like may be done for the halfe houres and quarters.

So also in the verticall declining plane. If you first take out the *secant* of the declination of the plane, and prick it downe in the horizontall line from A vnto E, and through E draw right lines parallell to the meridian, which will cut the former houre lines of 3 and 9, or one of them in the point C: then take out the semidiameter A V, and prick it downe in

those

those parallells from C vnto D, and draw right lines from A vnto C, and from V vnto D; the line V D shall be the houre of 6, and if you diuide these lines A C and D C, in such sort as you diuided the like line D C in the horizontall plane, you shall haue all the houre points required.

Or you may find the point D, in the houre of 6, without knowledge either of *H* or *C*. For hauing prickt downe *A V* in the meridian line, and *A E* in the horizontall line, and drawne parallels to the meridian through the points at *E*, you may take the *tangent* of the latitude out of the *Sector*, and fit it ouer in the sines of 90 and 90: so the parallell sine of the declination measured in the same *tangent* line, shall there shew the complement of the angle DVA, which the houre line of 6 maketh with the meridian; then hauing the point D, take out the semidiameter VA, and pricke it downe in those parallels from D vnto C: so shall you haue the lines D C and A C to be diuided as before.

The like might be vsed for the houre lines vpon all other planes. But I must not write all that may be done by the *Sector*. It may suffice that I haue wrote something of the vse of each line, and thereby giuen the ingenuous Reader occasion to thinke of more.

The conclusion to the Reader.

IT is well knowne to many of you, that this Sector was thus contriued, the most part of this booke written in Latin, many copies transcribed and dispersed more then sixteene yeares since. I am at the last contented to giue way that it come forth in English. Not that I thinke it worthy either of my labour or the publique view, but partly to satisfy their importunity, who not vnderstanding the Latin, yet were at the charge to buy the instrument, and partly for my owne ease. For as it is painefull for others to transcribe my copie, so it is troublesome for me to giue satisfaction herein to all that desire it. If I finde this to giue you content, it shall incourage me to do the like for my Crosse-staffe, *and some other Instruments. In the meane time beare with the Printers faults, and so I rest.*

Gresham Coll. 1. Maij. 1623. *E. G.*

FINIS.

Bridgewater ex dono Authoris.

THE DESCRIPTION AND VSE OF THE *CROSSE-STAFFE.*

For ſuch as are ſtudious of *Mathematicall practiſe.*

LONDON,
Printed by WILLIAM IONES.
and are to be ſold by IOHN TAP at Saint
Magnus corner. 1623.

THE

FIRST BOOKE OF THE *CROSSE-STAFFE.*

CHAP. I.

Of the description of the Staffe.

THe *Crosse-Staffe* is an instrument wel knowne to our Sea-men, and much vsed by the ancient Astronomers and others, seruing Astronomically for obseruation of altitude and angles of distance in the heauens, Geometrically for perpendicular heights and distances on land and sea.

The description and seuerall vses of it are extant in print, by *Gemma Frisius* in Latin, in English by *Dr. Hood.* I differ something from them both, in the proiection of this *Staffe,* but so, as their rules may be applied vnto it, and all their propositions be wrought by it: and therefore referring the Reader to their bookes, I shal be briefe in the explanation of that which may be applied from theirs vnto mine, and so come to the vse of those lines which are of my addition, not extant heretofore.

The necessary parts of this Instrument are fiue: the *Staffe,* the *Crosse,* and the three *sights.* The Staffe which I made for my owne vse, is a full yard in length, that so it may serue for measure.

The Croſſe belonging to it is 26 inches $\frac{1}{7}$ betweene the two outward ſights. If any would haue it in a greater forme, the proportion betweene the Staffe and the Croſſe, may be ſuch as 360 vnto 262.

The lines inſcribed on the Staffe are of foure ſorts. One of them ſerues for meaſure and protraction: one for obſeruation of angles: one for the Sea-chart; and the foure other for working of proportions in ſeuerall kinds.

The line of meaſure is an *inch line*, and may be knowne by his equall parts. The whole yard being diuided equally into 36 inches, and each inch ſubdiuided, firſt into ten parts, and then each tenth part into halfes.

The line for obſeruation of angles may be knowne by the double numbers ſet on both ſides of the line, beginning at the one ſide at 20, and ending at 90: on the other ſide at 40, and ending at 180: and this being diuided according to the degrees of a quadrant, I call it the *tangent line on the Staffe.*

The next line is the meridian of a Sea-chart, according to *Mercators* proiection from the Equinoctiall to 58 *gr.* of latitude, and may be knowne by the letter *M*, and the numbers 1.2. 3.4. vnto 58.

The lines for working of proportions, may be knowne by their vnequall diuiſions, and the numbers at the end of each line.

1 The line of *numbers* noted with the letter N, diuided vnequally into 1000 parts, and numbred with 1.2.3.4. vnto 10.

2 The line of *artificiall tangents* is noted with the letter *T*, diuided vnequally into 45 degrees, and numbred both ways, for the Tangent and the complement.

3 The line of *artificiall ſines*, noted with the letter *S*, diuided vnequally into 90 degrees, and numbred with 1.2.3.4. vnto 90.

4 The line of *verſed ſines* for more eaſie finding the houre and azimuth, noted with V, diuided vnequally into about 164 *gr.* 50 *m.* numbred backward with 10.20.30. vnto 164.

Thus there are ſeuen lines inſcribed on the Staffe: there are fiue lines more inſcribed on the Croſſe.

1 A Tangent line of 36 *gr.* 3 *m.* numbred by 5.10.15. vnto 35: the midst whereof is at 20 *gr*; and therefore I call it the *tangent of* 20; and this hath respect vnto 20 *gr.* in the Tangent on the Staffe.

2 A Tangent line of 49 *gr.* 6 *m.* numbred by 5.10.15. vnto 45; the midst whereof is at 30 *gr.* and hath respect vnto 30 *gr.* in the Tangent on the Staffe, whereupon I call it the *tangent of* 30.

3 A line of *inches* numbred with 1.2.3. vnto 26; each inch equally subdiuided into ten parts, answerable to the inch line vpon the Staffe.

4 A line of seuerall *chords*, one answerable to a circle of twelue inches semidiameter, numbred with 10.20.30. vnto 60: another to a semidiameter of a circle of six inches; and the third to a semidiameter of a circle of three inches; both numbred with 10.20.30. vnto 90.

5 A continuation of the *meridian* line from 57 *gr.* of latitude vnto 76 *gr*; and from 76 *gr.* to 84 *gr.*

For the inscription of these lines. The first for measure is equally diuided into inches and tenth parts of inches.

The tangent on the Staffe for obseruation of angles, with the tangent of 20 and the tangent of 30 on the Crosse, may all three be inscribed out of the ordinary *table of tangents*. The Staffe being 36 inches in length; the Radius for the tangent on the Staffe will be 13 inches and 103 parts of 1000: so the whole line will be a tangent of 70 *gr.* and must be numbred by their complements, & the double of their complements, the tangent of 10 *gr.* being numbred with 80 and 160.

The Radius for the tangent of 20 on the Crosse, will be 36 inches, and the whole line betweene the sights a tangent of 36 *gr.* 3 *m.* according as it is numbred. The Radius for the tangent of 30 *gr.* on the Crosse, will be 22 inches and 695 parts of 1000: so the whole line betweene the sights wil containe a tangent of 49 *gr.* 6 *m.* in such sort as they are numbred.

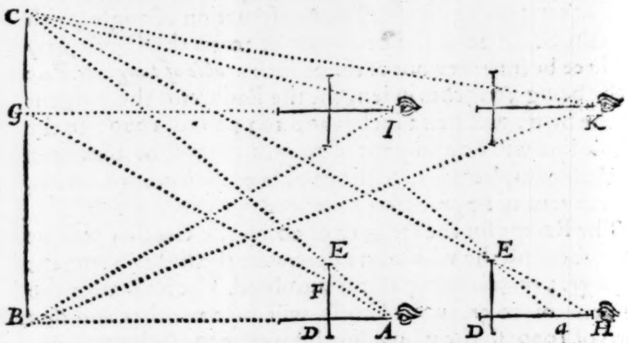
The meridian line may be inscribed out of the Table which I set downe for this purpose in the vse of the Sector.

The line of numbers may be inscribed out of the first Chiliad of Mr. *Briggs* Logarithmes: & the rest of the lines of proportion out of my *Canon of artificiall sines and tangents*; and in recompence thereof this booke will serue as a comment to explane the vse of my *Canon*.

CHAP. II.

The vse of the lines of inches *for perpendicular heights and distances.*

IN taking of heights and distances, the Staffe may be held in such sort, that it may be euen with the distance, and the Crosse parallell with the height: and then if the eye at the beginning of the Staffe shall see his marks by the inward sides of the two first sights, there will be such proportion betweene the distance and the height, as is betweene the parts intercepted on the Staffe and the Crosse. Which may be farther explaned in these propositions.



1 *To find an height at one station, by knowing the distance.*

Set the middle sight vnto the distance vpon the Staffe, the

the height will be found vpon the Crosse. For

As the segment of the Saffe
vnto the segment on the Crosse:
So is the distance giuen,
vnto the height.

As if the distance *A B* being knowne to be 256 feete, it were required to find the height *B C*: first I place the middle sight at 25 *inches* and 6 *parts* of 10, then holding the Staffe leuell with the distance, I raise the Crosse, parallell vnto the height, in such sort, as that my eye may see from *A* the beginning of the inches on the Staffe by the sight *E*, at the beginning of the inches on the Crosse vnto the mark *C*: which being done, if I find 19 inches and 2 parts of 10 intercepted on the Crosse betweene the sights at *E* and *D*, I would say the height *B C* were 192 feete.

Or if the obseruation were to be made before the distance were measured, I would set the middle sight either vnto 10 inches, or 12, or 16, or 20, or 24, or some such other number as might best be diuided into seuerall parts, and then worke by proportion. As if in the former example the middle sight were at 24 on the Staffe, and 18 on the Crosse, it should seem that the height is $\frac{3}{4}$ of the distance; and therefore the distance being 256, the height should be 192.

2 *To finde an height, by knowing some part of the same height.*

As if the height from *G* to *C* were knowne to be 48, and it were required to find the whole height *B C*: either put the third sight or some other running sight vpon the Crosse betweene the eye and the marke *G*. For then

As the difference betweene the sights,
vnto the whole segment of the Crosse:
So is the part of the height giuen,
vnto the whole height.

If then the difference betweene the sights *E* and *F*, shall

be 45, and the segment of the Crosse E D 180, the whole height *B C* will be found to be 192.

3 *To find an height at two stations, by knowing the difference of the same stations.*

As the difference of segments on the Staffe,
vnto the difference of stations:
So is the segment of the Crosse,
vnto the height.

Suppose the first station being at *H*, the segment of the Crosse *E D* were 180, and the segment of the Staffe *H D* 300: then coming 64 feete nearer vnto *B*, in a direct line, vnto a second station at *A*, and making another obseruation; suppose the segment of the Crosse *E D* were 180 as before, and the segment of the Staffe *A D* 240; take 240 out of 300, the difference of segments will be 60 parts. And

As 60 parts vnto 64 the difference of stations:
So D E 180 vnto *B C* 192 the height required.

In these three *Prop.* there is a regard to be had of the height of the eye. For the height measured, is no more then from the leuell of the eye vpward.

4 *To find a distance, by knowing the height.*

As the segment of the Crosse,
vnto the segment of the Staffe:
So is the height giuen,
vnto the distance.

So the segment E D being 18, and D A 24, the height *C B* 192, will shew the distance *A B* to be 256.

5 *To find a distance, by knowing part of the height.*

As the difference betweene the sights,
vnto the segment of the Staffe:

So

So is the part of the height giuen,
vnto the distance.

And thus the difference betweene *E* and *F* being 45, and the segment D A 240; the part of the height *GC* 48, will giue the distance *A* B to be 256.

6 *To finde a distance at two stations, by knowing the difference of the same stations.*

As the difference of segments on the Staffe,
vnto the difference of stations:
So is the whole segment,
vnto the distance.

And thus the segment of the Crosse being 180, the segment of the Staffe at the first station 240, at the second 300, the difference of the segments 60, & the difference of stations 64, the distance A B at the first station will be found to be 256, and the distance H B at the second station 320.

7 *To find a bredth by knowing the distance perpendicular to the bredth.*

This is all one with the first *Prop*. For this bredth is but an height turned sidewayes: and therefore
As the segment of the Staffe,
vnto the segment of the Crosse:
So is the distance
vnto the bredth.

And thus the segment of the Staffe being 24, and the segment of the Crosse 18, the distance A B 256, will giue the bredth B C to be 192.

8 *To find a bredth at two stations in a line perpendicular to the bredth, by knowing the difference of the same stations.*

This is also the same with the third *Prop*: and therefore

As the difference of segments on the Staffe,
vnto the difference of stations:
So the segment on the Crosse betweene the two sights,
vnto the bredth required.

And thus the difference betweene the stations at *A* and H being *64*, the difference of segments on the Staffe *60*, the segment of the Crosse 180, the bredth B C will be found to be 192.

In like maner may we finde the bredth *G C* for hauing found the bredth B C the proportion will hold.

As D E is vnto F E, so B *C* vnto *G C*. Or otherwise,
As H *a* vnto H A, so F E vnto *G C*.

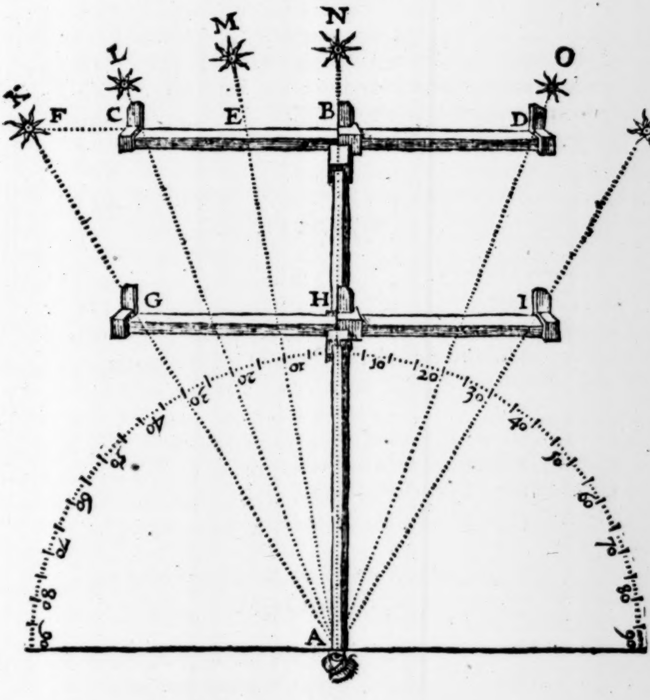
Neither is it materiall whether the two stations be chosen at the one end of the bredth proposed, or without it, or within it, if the line betweene the stations be perpendicular vnto the bredth: as may appeare if in stead of the stations at *A* and H, we make choise of the like stations at *I* and *K*.

There might be other wayes proposed to work these *Prop.* by holding the Crosse euen with the distance, and the *S*taffe parallell with the height: but these would proue more troublesome, and those which are deliuered are sufficient, and the same with those which others haue set downe vnder the name of the *Iacobs staffe.*

CHAP.

CHAP. III.

The vse of the Tangent lines in taking of Angles.



I *To find an angle by the Tangent on the Staffe.*

LEt the middle sight be alwayes set to the middle of the Crosse, noted with 20 and 30, and then the Crosse

drawne nearer the eye, vntill the marks may be seene close within the sights. For so if the eye at *A* (that end of the Staffe which is noted with 90 and 180) beholding the marks *K* and *N*, betweene the two first sights, *C* and *B*, or the marks *K* and *P* betweene the two outward sights, the Crosse being drawne downe vnto *H*, shall stand at 30 and 60, in the Tangent on the Staffe: it sheweth that the angle *K A N* is 30 *gr.* the angle *K A P* 60 *gr.* the one double to the other; which is the reason of the double numbers on this line of the Staffe: and this way wil serue for any angle from 20 *gr.* toward 90 *gr.* or from 40 *gr.* toward 180 *gr.* But if the angle be lesse then 20 *gr.* we must then make vse of the Tangent vpō the Crosse.

2 *To find an angle by the Tangent of* 20 *vpon the Crosse.*

Set 20 vnto 20, that is, the middle sight to the middest of the Crosse at the end of the Staffe, noted with 20: so the eye at *A*, beholding the marks *L* and *N*, close betweene the two first sights, *C* and *B*, shall see them in an angle of 20 *gr.*

If the marks shall be nearer together, as are *M* and *N*, then draw in the Crosse from *C* vnto *E*: if they be farther asunder, as are *K* and *N*, then draw out the Crosse from *C* vnto *F*; so the quantitie of the angle shal be still found in the Crosse in the Tangent of 20 *gr.* at the end of the Staffe; and this will serue for any angle from 0 *gr.* toward 35 *gr.*

3 *To find an angle by the Tangent of* 30 *vpon the Crosse.*

This Tangent of 30 is here put the rather, that the end of the Staffe resting at the eye, the hand may more easily remoue the Crosse: for it supposeth the Radius to be no longer then *A H*, which is from the eye at the end of the Staffe vnto 30 *gr.* about 22 *inches* and 7 *parts.* Wherefore here set the middle sight vnto 30 *gr.* on the Staffe, and then either draw the Crosse in or out, vntill the marks be seene between the

the two first sights; so the quantitie of the angle will be found in the Tangent of 30, which is here represented by the line *GH*; and this will serue for any angle from 0 *gr.* toward 48 *gr.*

4 *To obserue the altitude of the Sunne backward.*

Here it is fit to haue an horizontall sight set to the beginning of the Staffe, and then may you turne your backe toward the Sun, and your Crosse toward your eye. If the altitude be vnder 45 *gr.* set the middle sight to 30 on the Staffe, and looke by the middle sight through the horizontall vnto the horizon, mouing the Crosse vpward or downward, vntill the vpper sight doe shadow the vpper halfe of the horizontall sight: so the altitude will be found in the Tangent of 30.

If the altitude shal be more then 45 *gr.* set the middle sight vnto the middest of the Crosse, and look by the inward edge of the lower sight through the horizontall to the horizon, mouing the middle sight in or out, vntill the vpper sight do shadow the vpper halfe of the horizontall sight: so the altitude will be found in the degrees on the Staffe betweene 40 and 180.

5 *To set the Staffe to any angle giuen.*

This is the conuerse of the former *Prop.* For if the middle sight be set to his place and degree, the eye looking close by the sights as before, cannot but see his obiect in the angle giuen.

6 *To obserue the altitude of the Sunne another way.*

Set the middle sight to the middle of the Crosse, and hold the horizontall sight downward, so as the Crosse may be parallell to the horizon, then is the Staffe verticall; and if the outward sight of the Crosse do shadow the horizontall sight,

the complement of the altitude wil be found in the tangent on the Staffe.

7 *To obserue an altitude by thread and plummet.*

Let the middle sight be set to the middest of the Crosse, and to that end of the Staffe which is noted with 90 and 180; then hauing a thread and a plummet at the beginning of the Crosse, and turning the Crosse vpward, and the Staffe toward the Sunne, the thread will fall on the complement of the altitude aboue the horizon. And this may be applied to other purposes.

8 *To apply the lines of inches to the taking of angles.*

If the angles be obserued betweene the two first sights, there wil be such proportion between the parts of the Staffe and the parts of the Crosse, as betweene the Radius and the Tangent of the angle.

As if the parts intercepted on the Staffe were 20 inches, the parts on the Crosse 9 inches. Then by proportion as 20 vnto 9, so 100000 vnto 45000 the tangent of 24 *gr.* 14 *m.*

But if the angle shall be obserued betweene the two outward sights, the parts being 20 and 9 as before, the angle will be 48 *gr.* 28 *m.* double vnto the former.

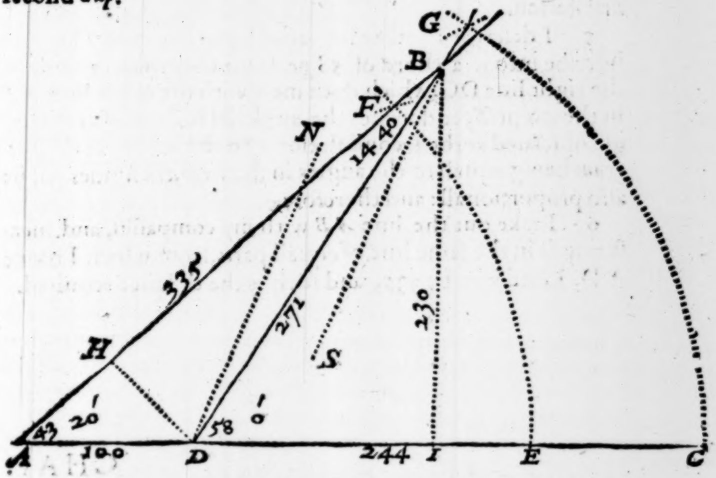
In all these there is a regard to be had to the parallax of the eye, and his height aboue the Horizon in obseruations at Sea; to the Semidiameter of the Sun, his parallax and refraction, as in the vse of other staues. And so this will be as much, or more then that which hath been heretofore performed by the *Crosse-staffe.*

CHAP.

CHAP. IIII.

The vse of the lines of equall parts ioyned with the lines of Chords.

THe lines of equall parts do serue also for protraction, as may appeare by the former *Diagrams*; but being ioyned with the lines of Chords, which I place vpon one side of the Crosse, they will farther serue for the protraction and resolution of right line triangles; whereof I will giue one example in finding of a distance at two stations otherwise then in the second *Cap.*



Let the distance required be *AB*. At *A* the first statiō I make choise of a station line toward *C*, and obserue the angle *BAC* by the tangent lines, which may be 43 *gr.* 20 *m*; then hauing gon an hundred paces toward *C*, I make my second station at D, where suppose I find the angle *BDC* to be 58 *gr.* or

the angle *BDA* to be 122 *gr*; this being done, I may finde the distance *A B* in this maner.

1 I draw a right line *A C*, representing the station line.

2 I take 100 out of the lines of equall parts, and pricke them downe from *A* the first station vnto D the second.

3 I open my compasses to one of the chords of 60 *gr.* and setting one foote in the point *A*, with the other I describe an occult arke of a circle intersecting the station line in *E*.

4 I take out of the same line of *chords* a chord of 43 *gr.* 20 *m.* (because such was the angle at the first station) and this I inscribe into that occult arke from *E* vnto *F*, which makes the angle F A D equall to the angle obserued at the first station.

5 I describe another like arke vpon the center D, and inscribe into it a chord of 58 *gr.* from *C* vnto *G*, and draw the right line DG, which doth meet with the other line *A F* in the point *B*, and makes the angle BDC equall to the angle obserued at the second station. So the angles in the *Diagram* being equall to the angles in the field, their sides wil be also proportionall: and therefore,

6 I take out the line *A B* with my compasses, and measuring it in the same line of equall parts, from which I tooke A D, I find it to be 335, and such is the distance required.

CHAP.

CHAP. V.

The vse of the Meridian line.

1 THe Meridian line, noted with the letter *M*, may serue for the more easie diuision of the plane sea-chart, according to *Mercators* proiection. For if you shall draw parallell meridians, each degree being halfe an inch distant from other, the degrees of this meridian line on the Staffe, shall giue the like degrees for the meridians on the chart, from the Equinoctiall toward the Pole: and then if through these degrees you draw streight lines perpendicular to the meridians, they shall be parallels of latitude.

If any desire to haue the degrees of his chart larger then those which I haue put on the Staffe, he may take these and increase them in a double, or treble, or a decuple proportion at his pleasure.

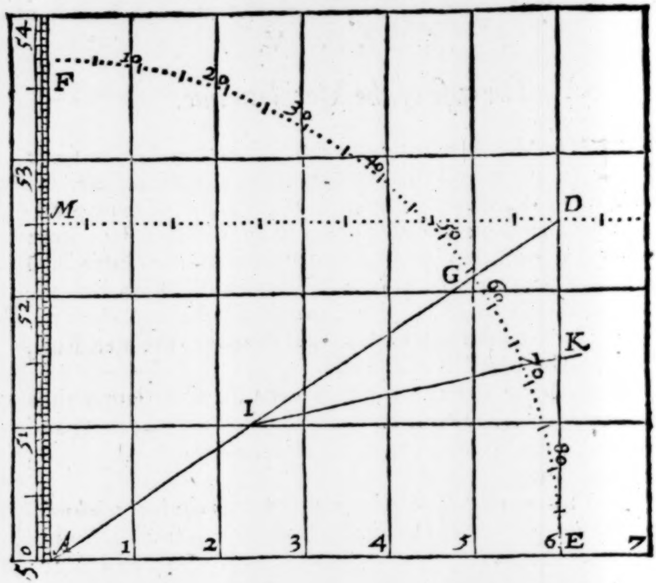
2 This *meridian* line being ioyned with the line of *chords*, may serue for the protraction & resolution of such right line triangles as concerne latitude, longitude, rumb and distance in the practise of nauigation. As may appeare by this example.

Suppose two places giuen, *A* in the latitude of 50 *gr.* D in the latitude of 52 *gr.* $\frac{1}{2}$, the difference of longitude betweene them being 6 *gr.* and let it be required to know, first what Rumb leadeth from the one place to the other; secondly how many degrees distant they are asunder.

1 I draw a right line *A E*, representing the parallell of the place from whence I depart.

2 I take 6 *gr.* for the difference of longitude, either out of the line of *inches*, allowing halfe an inch for euery degree, or out of the beginning of the *meridian* line; (for there the meridian degrees differ very little from the equinoctiall degrees) and these 6 *gr.* I pricke downe in the parallell from *A* to *E*.

3 In *A* and *E*, I erect two perpendiculars, AM and E D, representing the meridians of both places.



4 I take the difference of latitude from 50 *gr.* to 52 *gr.* 30 *m.* out of the ***meridian*** line, and pricke it downe in the meridians from *A* vnto *M*, and from E to D, and draw the right line MD for the parallell of the second place, and the right line AD for the line of distance betweene both places: so the angle MAD shall giue the Rumb that leadeth from the one place to the other.

5 To finde the quantitie of this angle MAD, I may either make vse of the Protractor, or else of a line of *chords*, and so I open my compasses vnto one of the chords of 60 *gr.* and setting one foote in the point *A*, with the other I describe an occult arke of a circle, intersecting the meridian in *F*, and the line of distance in G; then I take the chord FG with my compasses, and measuring it in the same line of *chords* as before, I find it 56 *gr.* $\frac{1}{4}$: and such is the inclination of the

the Rumb to the meridian, which is the first thing that was required.

6 To find the quantitie of the line of distance A D, I take it out with my compasses, and measuring it in the meridian line, setting one foote beneath the lesser latitude, and the other foote as much aboue the greater latitude, I find about 4 *gr.* $\frac{1}{2}$ intercepted betweene both feet: and such is the distance vpō the Rumb, which is the second thing that was required.

But if this example were protracted according to the common Sea-chart, where the degrees of the equinoctiall and meridian are both alike; the Rumb M A D would be found to be aboue 67 *gr.* and A D the distance vpon the Rumb about 6 *gr.* $\frac{1}{2}$.

Suppose farther, that hauing set forth from *A* toward D, vpon the former Rumb of 56 *gr.* 15 *m.* *N E b E*, after the ship had runne 36 leagues; the wind changing, it ran 50 leagues more vpon the seuenth Rumb of *E b N*, whose inclination to the meridian is 78 *gr.* 45 *m.* And let it be required to know what longitude and latitude the ship is in, by pricking down the way thereof vpon the Chart.

Hauing drawne a blanke chart as before, with meridians & parallels, according to the latitude of the places proposed,

1 I would make an angle M A D of 56 *gr.* 15 *m.* for the Rumb of *N E b E*, which is done after this maner: I open my compasses to one of the *chords* of 60 *gr.* and setting one foote in the point *A*, with the other I describe an occult ark of a circle, intersecting the meridian in *F*; then I take 56 *gr.* 15 *m.* out of the same line of *chords*, and pricke them downe from *F* vnto *G*: so the right line *A G* shall be the Rumb of *N E b E*.

2 I would take 36 leagues out of the *meridian line*, extending my compasses from 50 *gr.* to 51 *gr.* 48 *m.* or rather from as much below 50 as aboue 51; and prick them downe vpon the Rumb from *A* vnto *I*; so the point *I*, shal represent the place wherein the ship was when the wind changed. And this is in the latitude of 51 *gr.* 0 *m.* and in the longitude of 2 *gr.* 21 *m.* Eastward from the meridian *A M*.

3 By the same reason, I may draw the right line *I K* for the Rumb of *E b N*, and pricke downe the distance of 50 leagues from *I* vnto *K*: so the point *K* shal represent the place whither the ship came, after the running of these 50 leagues: and this is in the latitude of 51 *gr.* 30 *m.* and in longitude 6 *gr.* 16 *m.* Eastward from the first meridian *A M*, and therefore 16 *m.* Eastward from the second meridian E D.

But if these two courses were to be pricked downe by the common sea-chart, the point *I* would fall in the latitude of 51 *gr.* 0 *m.* and the point *K* in the latitude of 51 *gr.* 30 *m.* But the longitude of *I* would be onely 1 *gr.* 30 *m.* and the longitude of *K* onely 3 *gr.* 57 *m.* which is 33 *m* Westward from the meridian of the place to which the ship was bound.

Such is the difference betweene both these charts.

CHAP. VI.

The vse of the line of Numbers.

1 *Hauing two numbers giuen to find a third in continuall proportion, a fourth, a fift, and so forward.*

EXtend the compasses from the first number vnto the second; then may you turne them, from the second to the third, and from the third to the fourth, and so forward.

Let the two numbers giuen be 2 and 4. Extend the compasses from 2 to 4, then may you turne them from 4 to 8, and from 8 to 16, and from 16 to 32, and from 32 to 64, and from 64 to 128.

Or if the one foote of the compasses being set to 64, the other fall out of the line, you may set it to another 64 nearer the beginning of the line, and there the other foot will reach to 128, and from 128 you may turne them to 256, and so forward.

Or if the two first numbers giuen were 10 and 9: extend the compasses from 10 at the end of the line, backe vnto 9, then may you turne them from 9 vnto 8.1, and from 8.1

vnto

vnto 7.29. And so if the two first numbers giuen were 1 and 9, the third would be found to be 81, the fourth 729, with the same extent of the compasses.

In the same maner, if the two first numbers were 10 and 12, you may finde the third proportionall to be 14.4, the fourth 17.28. And with the same extent of the compasses, if the two first numbers were 1 and 12, the third would be found to be 144, and the fourth to be 1728.

2 *Hauing two extreme numbers giuen, to find a meane proportionall betw.en them.*

Diuide the space betweene the extreme numbers into two equall parts, and the foote of the compasses will stay at the meane proportionall. So the extreme numbers giuen being 8 and 32, the meane betweene them will be found to be 16, which may be proued by the former *Prop.* where it was shewed, that as 8 to 16, so are 16 to 32.

3 *To find the square roote of any number giuen.*

The square roote is alwayes the meane proportionall betweene 1 and the number giuen, and therefore to be found by diuiding the space betweene them into two equall parts. So the roote of 9 is 3, and the roote of 81 is 9, and the roote of 144 is 12.

4 *Hauing two extreme numbers giuen, to find two meane proportionals betweene them.*

Diuide the space betweene the two extreme numbers giuen, into three equall parts. As if the extreme numbers giuen were 8 and 27, diuide the space betweene them into three equall parts, the feet of the compasses will stand in 12 and 18.

5 *To find the cubique roote of a number giuen.*

The cubique roote is alwayes the first of two meane pro-

portionals betweene 1 and the number giuen, and therefore to be found by diuiding the space betweene them into three equall parts.

So the roote of 1728 will be found to be 12. The roote of 17280 is almost 26: and the roote of 172800 is almost 56.

6 To multiply one number by another.

Extend the compasses from 1 to the multiplicator; the same extent applied the same way, shall reach from the multiplicand to the product.

As if the numbers to be multiplied were 25 and 30: either extend the compasses from 1 to 25, and the same extent will giue the distance from 30 to 750; or extend them from 1 to 30, and the same extent shall reach from 25 to 750.

7 To diuide one number by another.

Extend the compasses from the diuisor to 1, the same extent shall reach from the diuidend to the quotient.

So if 750 were to be diuided by 25, the quotient would be found to be 30.

8 Three numbers being giuen to find a fourth proportionall.

This golden rule, the most vsefull of all others, is performed with like ease. For extend the compasses from the first number to the second, the same extent shall giue the distance from the third to the fourth.

As for example, the proportion between the diameter and the circumference, is said to be such as 7 to 22: if the diameter be 14, how much is the circumference? Extend the compasses from 7 to 22, the same extent shall giue the distance from 14 to 44: or extend them from 7 to 14, and the same extent shall reach from 22 to 44.

Either of these wayes may be tried on seuerall places of this

this line ; but that place is beſt, where the feete of the compaſſes may ſtand neareſt together.

9 *Three numbers being giuen to finde a fourth in a duplicated proportion.*

This propoſition concernes queſtions of proportion betweene *lines* and *ſuperficies*; where if the denomination be of lines, extend the compaſſes from the firſt to the ſecond number of the ſame denomination : ſo the ſame extent being doubled, ſhall giue the diſtance from the third number vnto the fourth.

The diameter being 14, the content of the circle is 154: the diameter being 28, what may the content be? Extend the compaſſes from 14 to 28, the ſame extent doubled will reach from 154 to 616. For firſt it reacheth from 154 vnto 308; and turning the compaſſes once more, it reacheth from 308 vnto 616 : and this is the content required.

But if the firſt denomination be of the ſuperficiall content, extend the compaſſes vnto the halfe of the diſtance, betweene the firſt number and the ſecond of the ſame denomination : ſo the ſame extent ſhall giue the diſtance from the third to the fouth.

The content of a circle being 154, the diameter is 14 : the content being 616, what may the diameter be ? Diuide the diſtance betweene 154 and 616 into two equall parts, then ſet one foote in 14, the other will reach to 28 the diameter required.

10 *Three numbers being giuen to find a fourth in a triplicated proportion.*

This propoſition concerneth queſtions of proportion betweene *lines* and *ſolids*; where if the firſt denomination be of lines, extend the compaſſes from the firſt number to the ſecond of the ſame denomination: ſo the extent being tripled, ſhall giue the diſtance frō the third number vnto the fourth.

Suppose the diameter of an iron bullet being 4 inches, the weight of it was 9 ℔: the diameter being 8 inches, what may the weight be? Extend the compasses from 4 to 8, the same extent being tripled, will reach from 9 vnto 72. For first it reacheth from 9 vnto 18; then from 18 to 36; thirdly from 36 to 72. And this is the weight required.

But if the first denomination shall be of the Solid content, or of the weight, extend the compasses to a third part of the distance betweene the first number and the second of the same denomination: so the same extent shal giue the distance from the third number vnto the fourth.

The weight of a cube being 72 ℔, the side of it was 8 inches: the weight being 9 ℔, what may the side be? Diuide the distance betweene 72 and 9, into three equall parts; then set one foote to 8, the other will reach to 4, the side required.

CHAP. VII.

The vse of the lines of artificiall Sines.

THis line of *sines* hath such vse in finding a fourth proportionall, as the ordinary *Canon* of *Sines*: and the maner of finding it, is alwayes such as in this example.

As the sine of 30 *gr.* vnto the sine of 52 *gr.*
So the sine of 38 *gr.* to a fourth sine.

Extend the compasses in the line of *sines* from 30 *gr.* vnto 52 *gr*; the same extent shall giue the distance from 38 *gr.* vnto 76 *gr.* Or extend them from 30 *gr.* vnto 38 *gr.* the same extent will reach from 52 *gr.* vnto 76 *gr.* which is the fourth proportionall sine required.

And thus may the rest of all sinical proportions be wrought two wayes. The minutes which are wanting in the first degree, may be supplied by the line of *Numbers*.

CHAP.

CHAP. VIII.

The vse of the line of artificiall Tangents.

THis line of *Tangents* hath like vſe, but commonly ioyned with the line of *ſines*: the maner of working by it, may appeare by this example.

As the Tangent of 38 *gr.* 30 *m.*
is to the Tangent of 23 *gr.* 30 *m.*
So the Sine of 90 *gr.*
to a fourth Sine.

This *Prop.* and ſuch others vpon two lines, may be wrought two wayes. For extend the compaſſes from the Tangent of 38 *gr.* 30 *m.* to the Tangent of 23 *gr.* 30 *m*; the ſame extent ſhall giue the diſtance from the ſine of 90 *gr.* to the ſine of 33 *gr.* 8 *m.* Or elſe extend them from 38 *gr.* 30 *m.* in the Tangents vnto 90 *gr.* in the line of *ſines*; the ſame extent from the Tangent of 23 *gr.* 30 *m.* ſhall reach to the ſine of 33 *gr.* 8 *m.* which is the fourth proportionall ſine required.

And this croſſework in many caſes is the better, in regard the tangents which ſhould paſſe on from 40 *gr.* to 50 *gr.* and ſo forward, do turne backe at 45 *gr.* Theſe two lines of *Sines* and *Tangents*, may ſerue for the reſolution of all ſphericall triangles, according to thoſe Canons which I haue ſet downe in the vſe of the *Sector*.

Or if at any time one meete with a *ſecant*, let him account the ſine of 80 *gr.* for a *ſecant* of 10 *gr.* and the ſine of 70 *gr.* for a *ſecant* of 20 *gr.* and ſo take the ſine of the complement in ſtead of the *ſecant*. As if the propoſition were,

As the Radius to the ſecant of 51 *gr.* 30 *m.*
So the ſine of 23 *gr.* 30 *m.* to a fourth ſine.

Extend the compaſſes from the Radius that is the ſine of 90 *gr.* to the ſine of 38 *gr.* 30 *m.* the ſame extent will giue the diſtance from the ſine of 23 *gr.* 30 *m.* both to the ſine of 14 *gr.*

22

22 *m.* and to the sine of 39 *gr.* 50 *m.* But in this case, the sine of 39 *gr.* 50 *m.* is the fourth required. For the first number being lesse then the second, that is, the Radius lesse then the secant, the sine of 23 *gr.* 30 *m.* which is the third, must also be lesse then the fourth.

CHAP. IX.

The vse of the line of Sines and Tangents ioyned with the line of numbers.

THe lines of *sines* and *tangents* haue another like vse ioyned with the line of *numbers*, especially in the resolution of right line triangles, where the angles are measured by degrees and minutes, and the sides measured by absolute numbers, whereof I will set downe these propositions.

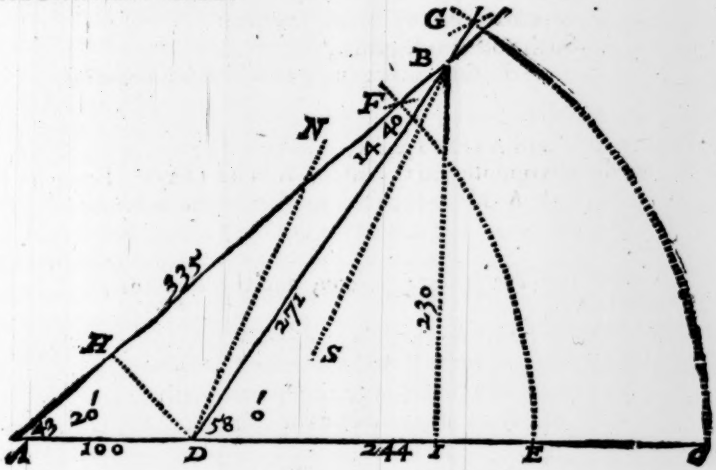
1 *Hauing three angles and one side, to find the two other sides.*

As the sine of the angle opposite to the side giuen,
is to the number belonging to that side giuen:
So the sine of the angle opposite to the side required,
to the number belonging to the side required.

As in the example of the fourth *Cap.* of this booke, where knowing the distance betweene two stations at *A* and *D* to be 100 paces, the angle *B A C* to be 43 *gr.* 20 *m.* and the angle *B D C* to be 58 *gr.* it was required to find the distance *A B.*

First hauing these two angles, I may find the third angle *A B D* to be 14 *gr.* 40 *m.* either by substraction or by complement vnto 180. Then in the triangle *B A D*, I haue three angles, and one side, whereby I may find both *A B* and *D B.* I know the angle *A B D* opposite to the measured side *A D* to be 14 *gr.* 40 *m.* and the angle *A D B* opposite to the side required, to be 122 *gr:* wherefore I extend the compasses in the

the line of *sines* from 14 *gr.* 40 *m.* vnto 122 *gr.* or (which is all one) to 58 *gr.* (for after 90 *gr.* the sine of 80 *gr.* is also the sine of 100 *gr.* and the sine of 70 *gr.* the sine of 110 *gr.* and so in the rest) so shall I find the same extent to reach in the line of *numbers*, from 100 vnto 335. And such is the distance requi-red betweene *A* and *B*.



In like maner if I extend my compasses from the sine of 14 *gr.* 40 *m.* to the sine of 43 *gr.* 20 *m.* the same extent will reach in the line of *numbers* from 100 to 271. And such is the distance betweene D and B.

Or in crosse worke, I may extend the compasses from 14 *gr.* 40 *m.* in the *sines*, vnto 100 parts in the line of *numbers*: so the same extent will giue the distance from 58 *gr.* to 335 parts, and from 43 *gr.* 20 *m.* to 271 parts.

2 *Hauing two sides giuen, and one angle opposite to either of these sides, to find the other two angles and the third side.*

As the side opposite to the angle giuen,
is to the sine of the angle giuen:
So the other side giuen,
to the sine of that angle to which it is opposite.

So in the former triangle, hauing the two sides *AB* 335 paces, and A D 100 paces, and knowing the angle A D B, which is opposite to the side *AB*, to be 122 *gr.* I may find the angle A B D, which is opposite to the other side A D. For if I extend the compasses from 335 to 100 in the line of *numbers*, I shall finde the same extent to reach in the line of *sines* from 122 *gr.* to 14 *gr.* 40 *m*; and therefore such is the angle A B D.

Then knowing these two angles ABD and ADB, I may find the third angle B A D either by subtraction or by complement to 180, to be 43 *gr.* 20 *m*; and hauing three angles and two sides, I may well find the third side DB, by the former *Prop.*

This may be done more readily by crosse worke. For if I extend the compasses from 335 parts, in the line of *numbers*, to the sine of 122 *gr.* the same extent wil reach from 100 parts to the sine of 14 *gr.* 40 *m.* and backe from 43 *gr.* 20 *m.* to 271 parts; and such is the third side DB.

3 *Hauing two sides and the angle betweene them, to find the two other angles and the third side.*

If the angle contained betweene the two sides be a right angle, the other two angles will be found readily by this canon.

As the greater side giuen,
is to the lesser side:

So-

So the tangent of 45 *gr.*
to the tangent of the lesser angle.

So in the rectangle triangle *A I B*, knowing the side *A I* to be 244, and the side *I B* to be 230: if I extend the compasses from 244 to 230 in the line of *numbers*, the same extent will reach from 45 *gr.* to about 43 *gr.* 20 *m.* in the line of *tangents*; and such is the lesser angle *B A I*, and the complement 46 *gr.* 40 *m.* shewes the greater angle *A B I*. The angles being knowne, the third side A B may be found by the first *Prop.*

So likewise in the example of the third *Cap.* of this booke, concerning taking of angles by the line of *inches*, where the parts intercepted on the Staffe being 20 *inches*, and the parts on the Crosse 9 *inches*, it was required to find the angle of altitude. For I may extend the compasses in the line of *numbers*, from 20 vnto 9, the same extent will reach in the line of *tangents*, from 45 *gr.* to 24 *gr.* 14 *m.* Or in the crosse worke, I may extend the compasses from 20 parts in the line of *numbers* to the tangent of 45 *gr*; the same extent shall giue the distance from 9 parts vnto the tangent of 24 *gr.* 14 *m.* And such is the angle of altitude required.

But if it be an oblique angle that is contained betweene the two sides giuen, the triangle may be reduced into two rectangle triangles, and then resolued as before.

As in the triangle A D B, where the side A B is 335, and the side A D 100, and the angle B A D 43 *gr.* 20 *m*: if I let downe the perpendicular DH vpon the side A B, I shal haue two rectangle triangles, AHD, DHB; and in the rectangle AHD, the angle at *A* being 43 *gr.* 20 *m.* the other angle ADH will be 46 *gr.* 40 *m*; and with these angles and the side *A D*, I may find both *A H* and D*H*, by the first *Prop.* Then taking *A H* out of *A B*, there remaines H B for the side of the rectangle DHB; and therefore with this side H B and the other side D H, I may find both the angle at B, and the third side D B, as in the former part of this *Prop.*

Or I may find the angles required, without letting downe any perpendicular. For

As the summe of the sides,
is to the difference of the sides:
So the tangent of the halfe summe of the opposite angles,
to the tangent of half the difference between those angles.

As in the former triangle *A D B*, the summe of the sides *A B*, *A D*, is 435, and the difference betweene them 235; the angle contained 43 *gr*. 20 *m*; and therefore the summe of the two opposite angles 136 *gr*. 40 *m*. and the halfe summe 68 *gr* 20 *m*. Hereupon I extend the compasses in the line of *numbers* from 455 to 235, and I find them to reach in the line of *tangents* from 68 *gr*. 20 *m*. vnto 53 *gr*. 40 *m*; and such is the halfe difference betweene the opposite angles at *B* and *D*. This halfe difference being added to the halfe summe, doth giue 122 *gr*. for the greater angle *A D B*: and being subtracted, it leaueth 14 *gr*. 40 *m*. for the lesser angle *A B D*. Then the three angles being knowne, the third side B D may be found by the first *Prop.*

4 *Hauing the three sides of a right line triangle, to find the perpendicular and the three angles.*

Let one of the three sides giuen be the base, but rather the greater side, that the perpendicular may fall within the triangle; then gather the summe, and the difference of the two other sides, and the proportion will hold.

As the base of the triangle,
is to the summe of the sides:
So the difference of the sides
to a fourth, which being taken forth of the base, the perpendicular shal fall on the middle of the remainder.

As in the former triangle ADB, where the base AB is 335, the summe of the sides AD and DB 371, and the difference of them 171. If I extend the compasses in the line of *numbers* from 335 vnto 371, I shall find the same extent to reach from 171 vnto 189.4. This fourth number I take out of the base

base 335.0, and the remainder is 145.6, the halfe whereof is 72.8, and doth shew the place *H*, where the perpendicular shall fall, from the angle D, vpon the base *A B*, diuiding the former triangle ADB into two right angle triangles, DHA and DHB, in which the angles may be found by the former part of the third *Prop.* And this may suffice for right line triangles. But for the more easie protraction of these triangles, I will set downe one proposition more concerning *chords.*

5 *Hauing the semidiameter of a circle, to find the chords of euery arke.*

As the sine of 30 *gr.*
to the sine of halfe the arke proposed:
So is the semidiameter of the circle giuen,
to the chord of the same arke.

As if in protracting the former triangle ADB, it were required to find the length of a chord of 43 *gr.* 20 *m.* agreeing to the semidiameter *A E*, which is knowne to be 3 *inches.* The halfe of 43 *gr.* 20 *m.* is 21 *gr.* 40 *m*; wherefore I extend the compasses from the sine of 30 *gr.* to the sine of 21 *gr.* 40 *m.* and I finde the same extent to reach in the line of *numbers* from 3.000 *parts* to 2.215; which shewes, that the semidiameter being 3 inches, the chord of 43 *gr.* 20 *m.* will be 2 *inches* and 215 *parts* of 1000.

In like maner the chord of 58 *gr.* agreeing to the same semidiameter, would be found to be 2 *inches* and 909 *parts.* For the halfe of 58 being 29; if I extend the compasses in the line of *sines* from 30 *gr.* to 29 *gr.* the same extent will reach in the line of *numbers* from 3.000 vnto 2.909.

Or in crosse worke, if I extend the compasses from the *sine* of 30 *gr.* to 3.000 in the line of *numbers*, I shall find the same extent to reach from 21 *gr.* 40 *m.* to 2.215 *parts*, and from 29 *gr.* to 2.909 *parts*, and from 7 *gr.* 20 *m.* to 765 *parts*; for the chord of 14 *gr.* 40 *m.* for the third angle ABD.

CHAP. X.

The vse of the line of versed sines.

THis line of *versed sines* is no necessary line. For all triangles, both right lined and sphericall, may be resolued by the three former lines of ***numbers***, *sines* and ***tangents***; yet I thought good to put it on the Staffe for the more easie finding of an angle hauing three sides, or a side hauing three angles of a sphericall triangle giuen.

Suppose the three sides to be, one of them 110 *gr.* the other 78 *gr.* and the third 38 *gr.* 30 *m.* and let it be required to find the angle, whose base is 110 *gr.*

I first adde them together, and from halfe the summe subtract the base, noting the difference after this maner.

The base	110 *gr.*	0 *m.*
The one side	78	0
The other side	38	30
The summe of all three	226	30
The halfe summe	113	15
The difference	3	15

This done, I come to the Staffe, and extend the compasses from the sine of 90 *gr.* to the sine of 78 *gr.* which is one of the sides; and applying this extent from the sine of the other side 38 *gr.* 30 *m.* I find it to reach to a fourth sine, about 37 *gr.* 30 *m.* From this fourth sine of 37 *gr.* 30 *m.* I extend the compasses again, to the sine of the halfe summe 113 *gr.* 15 *m.* (which is all one with the sine of 66 *gr.* 45 *m.*) and this second extent wil reach from the sine of the difference 3. *gr.* 15 *m.* to the sine of 4 *gr.* 54 *m.* Ouer against this sine you shal find 146 *gr.* in the line of *versed sines*; and such is the angle required.

THE

THE SECOND BOOKE.

Of the vſe of the former lines of proportiō, more particularly exemplified in ſeuerall kinds.

THe former booke containing the generall vſe of each line of proportion, may be ſufficient for all thoſe which know the rule of *Three*, and the doctrine of triangles.

But for others, I ſuppoſe it would be more difficult to find either the declination of the Sunne, or his amplitude, or the like, by that which hath been ſaid in the vſe of the line of *ſines*, vnleſſe they may haue the particular proportions, by which ſuch propoſitions are to be wrought. And therefore for their ſakes I haue adioyned this ſecond booke, containing ſeuerall proportions for propoſitions of ordinary vſe, and ſet them down in ſuch order, that the Reader conſidering which is the firſt of the three numbers giuen, may eaſily apply them to the Sector, and alſo reſolue them by Arithmetique, beginning with thoſe which require help onely of the line of *numbers*.

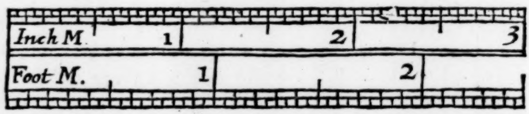
CHAP. I.

The vſe of the line of Numbers in broade meaſure, ſuch as boord, glaſſe, and the like.

THe ordinary meaſure for bredth and length are feete and inches, each foote diuided into 12 inches, and euery inch into halues & quarters,

ters, which being parts of ſeueral denominations, doth breed much trouble both in arithmetique and the vſe of inſtruments.

For the auoiding whereof, where I may preuaile I giue this counſell, that ſuch as are delighted in meaſure would vſe ſeuerall lines, firſt a line of inch meaſure, wherein euery inch may be diuided into 10 or 100 parts; ſecondly a line of foote meaſure, wherein euery foote may be diuided into 100 or 1000 parts, both which lines may be ſet on the ſame ſide of a two foote ruler, after this or the like maner.



Then if they be to giue the content of any ſuperficies or ſolid in inches, they may meaſure the ſides of it by the line of inches and parts of inches; but if they be to giue the content in feete, it would be more eaſie for them to meaſure thoſe ſides by the foote line and his parts.

For example, let the length of a plane be 30 inches, and the bredth 21 inches and $\frac{6}{10}$ of an inch; this length multiplied into the bredth, would giue the content to be 648 inches: but if I were to find the content of the ſame plane in feet, I would meaſure the ſides of it by the foote line and his parts; ſo the length would proue to be 2 feete $\frac{50}{100}$, and the bredth 1 foote $\frac{80}{100}$, and the length multiplied by the bredth, cutting off the foure laſt figures, for the foure figures of the parts, would giue the content to be 4.5000, which is 4 foote and 5000 parts, of a foot being diuided into 10000 parts.

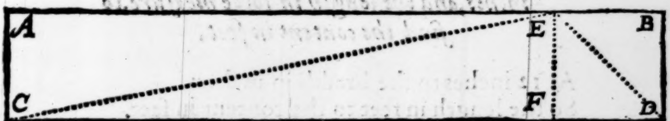
$$\begin{array}{r} 21.6 \\ \underline{30.0} \\ 648.00 \end{array} \qquad \begin{array}{r} 2.50 \\ \underline{1.80} \\ 20000 \\ \underline{250} \\ 4.5000 \end{array}$$

The like reason holdeth for yards and elnes, and all other measures diuided into 10,100,or 1000 parts.

This being presupposed, the worke will be more easie both by arithmetique and the line of *numbers*, as may appeare by these propositions.

1 *Hauing the bredth and length of any oblong superficies giuen in inch-measure, to finde the content in inches.*

As 1 inch vnto the bredth in inches:
So the length in inches vnto the content in inches.



Suppose in the plane AD, the bredth *A C* to be 30 inches, and the length *A B* to be 183 inches; extend the compasses from 1 vnto 30, the same extent will reach from 183 vnto 5490; or extend them from 1 vnto 183, the same extent will reach from 30 vnto 5490. So both wayes the content required is found to be 5490 inches.

As 1 vnto 30: so are 183 vnto 5490.

2 *Hauing the length and bredth of any oblong superficies giuen in inches, to finde the content in feete.*

As 144 inches vnto the bredth in inches:
So the length in inches vnto the content in feete.

And thus in the former plane A D, working as before, the content will be found to be 38.125, which is 38 foote and $\frac{1}{8}$ of a foote.

As 144 vnto 30: so are 183 vnto 38.125.

3 *Hauing the length and bredth of any oblong superficies giuen in foote measure, to finde the content in feete.*

As 1 foote vnto the bredth in foote measure:
So the length in feete vnto the content in feete.

And thus in the former plane AD, the bredth will be 2 foote 50 parts, and the length 15 foot 25 parts; then working as before, the content will be found to be 38.125.

As 1 vnto 2.50 : so are 15.25 vnto 38.125.

4 *Hauing the bredth of any oblong superficies giuen in inches, and the length in foote measure, to find the content in feet.*

As 12 inches to the bredth in inches:
So the length in feet to the content in feet.

So also in the former plane, the content will be found to be 38.125.

As 12 vnto 30: so are 15.25 vnto 38.125.

5 *Hauing the bredth of an oblong superficies giuen in inches, to find the length of a foot superficiall in inch measure.*

As the bredth in inches, vnto 144 inches:
So 1 foot vnto the length in inch measure.

So the bredth being 30 inches, the length of a foot wil be found to be 4 inches 80 parts.

As 30 vnto 144 : so are 1 vnto 4.80.

6 *Hauing the bredth and length of an oblong superficies giuen in feet, to find the length of a foot superficiall in foot measure.*

As the bredth in foot measure to 1 foot:
So the number of feet to the length in foot measure.

So

So the bredth being 2 foot 50 parts, the length of a foot will be found to be 40 parts, the length of 2 feete 80 parts, and the length of 3 feet 1 foot 20 parts, &c.

As 250 vnto 1: so are 1 vnto 0.40.

7 *Hauing the length and bredth of an oblong superficies, to find the side of a square equall to the oblong.*

Diuide the space betweene the length and the bredth into two equall parts, and the foot of the compasses will stay at the side of the square.

So the length being 183 inches, and the bredth 30 inches, the side of the square will be found to be almost 74 inches and 10 parts of 100.

Or the bredth being 2 foot and 50 parts, the length 15 foot and 25 parts, the side of the square wil be found to be about 6 feet and 17 parts.

As 30 vnto 74.10: so are 74.10 vnto 183.027.
And as 2.50 vnto 6.174: so are 6.174 vnto 15.247.

8 *Hauing the diameter of a circle, to find the side of a square equall to that circle.*

As 10000 to the diameter:
So 8862 vnto the side of the square.

So the diameter of a circle being 15 inches, the side of the square will be found about 13 inches and 29 parts.

As 10000 vnto 8862: so are 15 vnto 13.29.

9 *Hauing the circumference of a circle to find the side of a square equall to the same circle.*

As 10000 to the circumference:
So 2821 to the side of the square.

So the circumference of a circle being 47 inches 13 parts, the side of the square will be about 13 inches 29 parts.

As 10000 vnto 2821: so are 47.13 vnto 13.29.

10 *Hauing the diameter of a circle, to find the circumference.*

11 *Hauing the circumference of a circle, to find the diameter.*

As 1000 to the diameter:
So 3142 to the circumference.

So the diameter being 15 inches, the circumference will be found about 47 inches 13 parts: or the circumference being 47.13, the diameter will be 15.

CHAP. II.

The vse of the line of Numbers in the measure of land by perches and acres.

1 *Hauing the bredth and length of an oblong superficies giuen in perches, to find the content in perches.*

As 1 perch to the bredth in perches:
So the length in perches to the content in perches.

So in the former plane *AD*, if the bredth *AC* be 30 perches, and the length *AB* 183 perches, the content will be found to be 5490 perches.

2 *Hauing the length and bredth of an oblong superficies giuen in perches, to find the content in acres.*

As 160 to the bredth in perches:
So the length in perches to the content in acres.

So in the former plane *AD*, the content will be found to be 34 acres, and 31 centesms or parts of an 100.

As 160 vnto 30: so are 183 vnto 34.31.

3 *Hauing the length and bredth of an oblong ſuperficies giuen in chaines, to find the content in acres.*

It being troubleſome to diuide the content in perches by 160, we may meaſure the length and bredth by chaines, each chaine being 4 perches in length, and diuided into 100 links, then will the worke be more eaſie in arithmetique. For

As 10 to the ~~content~~ breadth in chaines:
So the length in chaines to the content in acres.

And thus in the former plane AD, the bredth *A C* will be 7 chaines 50 links, and the length *A B* 45 chaines 75 links; then working as before, the content will be found as before, 34 acres 31 parts.

4 *Hauing the perpendicular and baſe of a triangle giuen in perches, to find the content in acres.*

If the perpendicular go for the bredth, and the baſe for the length, the triangle will be the halfe of the oblong. As the triangle C E D is the halfe of the oblong AD, whoſe content was found in the former *Prop.* Or without halfing,

As 320 to the perpendicular:
So the baſe to the content in acres.

So in the triangle C E D, the perpendicular being 30, and the baſe 183, the content will be found to be about 17 acres and 15 parts.

5 *Hauing the perpendicular and baſe of a triangle giuen in chaines, to find the content in acres.*

As 20 to the perpendicular:
So the baſe to the content in acres.

And ſo in the triangle C E D, the perpendicular *E F* be-

ing 7.50, and the base C D 45.75, the content will be found as before to be about 17 acres 15 parts.

6 *Hauing the content of a superficies after one kind of perch, to find the content of the same superficies according to another kind of perch.*

As the length of the second perch
to the length of the first perch:
So the content in acres to a fourth number;
and that fourth to the content in acres required.

Suppose the plane A D measured with a chaine of *66* feet, or with a perch of *16* feete and an halfe, contained 34 acres 31 parts; and it were demanded how many acres it would containe if it were measured with a chaine of 18 foot to the perch: these kind of propositions are wrought by the backward rule of *three*, after a duplicated proportion. Wherefore I extend the compasses from *16.5* vnto 18.0, and the same extent doth reach backward, first from 34.31 to 31.45, and then from 31.45 to 28.84, which shewes the content to be 28 acres 84 parts.

7 *Hauing the plot of a plane with the content in acres, to find the scale by which it was plotted.*

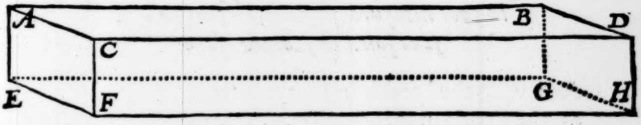
Suppose the plane A D contained 34 acres 31 centesmes; if I should measure it with a scale of 10 in the inch, the length AB would be 38 chaines and about 12 centesmes, and the bredth *AC 6* chaines and 25 centesmes; and the content would be found by the third *Prop.* of this Chapter, to be about 23 acres 82 parts, whereas it should be 34 acres 31 parts.

Wherefore I diuide the distance betweene 23.82, and 34.31, vpon the line of *numbers* into two equall parts; then setting one foote of the compasses vpon 10, my supposed scale, I find the other to extend to 12, which is the scale required.

CHAP.

CHAP. III.

The vse of the line of Numbers in solid measure, such as stone, timber, and the like.



1 *Hauing the side of a square equall to the base of any solid giuen in inch measure, to find the length of a foot solid in inch measure.*

THe side of a square equall to the base of a solid, may be found by diuiding the space betweene the length and bredth into two eqnall parts, as in the 7 *Prop.* of broad measure. Then

As the side of the square in inches to 41.57:
So is 1 foot to a fourth number;
and that fourth to the length in inches.

So in the solid *A H*, the side of the square equall to the base *E C*, being about 25 inches 45 parts, the length of a foot solid will be found about 2 inches 67 parts, and the length of two foot solid 5 inches 33 parts.

As 25.45 vnto 41.57 : so 1.00 vnto 1.63:
and so are 1.63 vnto 2.67.

2 *Hauing the side of a square equall to the base of any solid giuen in foot measure, to find the length of a foot solid in foot measure.*

As the side of the square in feet vnto 1:
So is 1 vnto a fourth number;
And that fourth to the length in foot measure.

So in the solid *A H*, the side of the square equall to the base

base *E C*, being about 2 foote 120 parts, the length of a foot solid will be found about 222 parts of a foot.

As 2.120 vnto 1.000: so 1.000 vnto 0.471:
and so are 471 vnto 222.

3 *Hauing the bredth and depth of a squared solid giuen in foot measure, to find the length of a foot solid in foot measure.*

As 1 vnto the bredth in foot measure:
So the depth in feet to a fourth number;
which is the content of the base in foot measure. Then
As this fourth number vnto 1:
So 1 vnto the length in foot measure.

So in the solid *A H*, the bredth being 2 foot 50 parts, the depth 1 foot 80 parts, the content of the base *E C* will be found 4 foot 50 parts, and the length of one foot solid about 222 parts, the length of two foot solid about 444 parts of 1000.

As 1.00 vnto 2.50: so are 1.80 vnto 4.50.
As 4.50 vnto 1.00: so 1.000 vnto 0.222.

4 *Hauing the bredth and depth of a squared solid giuen in inches, to find the length of a foot solid in inch measure.*

As 1 hath to the bredth in inches:
So the depth in inches to a fourth number;
which is the content of the base in inches. Then
As this fourth number vnto 1728:
So 1 vnto the length of a foot in inch measure.

So in the solid *A H*, the bredth *A C* being 30 inches, and the depth *A E* 21 inches 60 parts, the content of the base *E C* will be found to be 648 inches, and the length of a foot solid about 2 inches 67 parts.

As 1 vnto 21.6: so 30 vnto 648:
As 648 vnto 1728: so 1 vnto 2.667.

Or as 12 to the bredth in inches:
So the depth in inches to a fourth number.

As this fourth number to 144:
So 1 vnto the length of a foot solid in inch measure.

So in the solid *AH*, the bredth being 30 inches, the depth 21 inches 6 parts, the fourth number will be found to be 54, and the depth of a foot solid 2 inches 67 parts.

As 12 vnto 21.6: so 30 vnto 54.
As 54 vnto 144: so 1 vnto 2.667.

5 *Hauing the side of a square equall to the base of any solid, and the length thereof giuen in inch measure, to find the content thereof in feet.*

As 41.57 to the side of the square in inches:
So the length in inches to a fourth number;
and that fourth to the content in foot measure.

So in the solid *AH*, the length *AB* being 183 inches, and the side of the square equall to the base *EC* about 25 inches 45 parts, the fourth number will be found about 112, and the whole solid content about 68 feet 62 parts.

As 41.57 vnto 25.45: so 183 vnto 112:
and so are 112 vnto 68.62.

6 *Hauing the side of a square equall to the base of any solid, and the length thereof giuen in foot measure, to find the content thereof in feet.*

As 1 to the side of the square in foot measure:
So the length in feet to a fourth number;
and that fourth to the content in foot measure.

So in the former solid *A H*, the side of the square equall to the base *A E*, being about 2 foot 12 parts, and the length *A B* 15 foot 25 parts, the conent will be found to be about 68 foot 62 parts.

As 1 vnto 2.12: so 15.25 vnto 32.35:
and so are 32.35 vnto 68.62.

7 *Hauing the side of a square equall to the base of any solid giuen in inch measure, & the length of the solid giuen in foot measure, to find the content thereof in feete.*

As 12 to the side of the square giuen in inches:
So the length in feet to a fourth number;
and that fourth to the content in foot measure.

So in the former solid *A H*, the side of the equall square being 25 inches 45 parts, the content will be found to be a-about 68 feet 62 parts.

As 12 vnto 25.45: so 15.25 vnto 32.35:
and so are 32.35 vnto 68.62.

8 *Hauing the length, bredth and depth of a squared solid giuen in inches, to find the content in inches.*

As 1 vnto the bredth in inches:
So the depth in inches vnto the base in inches. Then
As 1 vnto the base:
So the length in inches vnto the solid content in inches.

So in the solid *A H*, whose bredth *A C* is 30 inches, the depth *A E* 21 inches and 6 parts of 10, and length *A B* 183, the content of the base *E C* wil be found 648 inches, and the whole solid content about 118500 inches.

As 1 vnto 21.6: so are 30 vnto 648:
As 1 vnto 648: so are 183 to 118584.

9 *Hauing the length, bredth and depth of a squared solid giuen in inches, to find the content in feet.*

As 1 to the bredth in inches:
So the depth in inches to the base in inches.

As 1728 to that base:
So the length in inches to the content in feet.

So in the solid *A H*, the content will be found to be about *68* feet *62* parts.

As 1 vnto 21.*6*: so 30 vnto *648*:
As 1728 vnto *648*: so 183 to *68.62*.

Or as 12 to the bredth in inches:
So the depth in inches to a fourth number.
As 144 to that fourth number:
So the length in inches to the content in feet.

And so also in the same solid A H, the content will be found to be about *68* feet *62* parts.

As 12 vnto 21.*6*: so 30 vnto 54:
As 144 vnto 54: so 183 vnto *68.62*.

10 *Hauing the length, bredth and depth of a squared solid giuen in foot measure, to finde the content in feet.*

As 1 vnto the bredth in foot measure:
So the depth in feet to the base in feet.

As 1 vnto that base:
So the length in feet to the content in feet.

And thus in the former solid A H, the bredth *A C* will be 2 foot 50 parts, the depth *A E* 1 foot 80 parts, and the length *A B* 15 foot 25 parts; then working as before, the content of the base *A F* will be found 4 feet 50 parts, and the whole solid content about 68 foot *62* parts, which of all others may

 very

very easily be tried by arithmetique.

As 1 vnto 2.50: so 1.80 vnto 4.50.
As 1 vnto 4.50: so 15.25 vnto 68.625.

11 *Hauing the bredth and depth of a squared solid giuen in inches, and the length in foot measure, to find the content thereof in feet.*

As 1 vnto the bredth in inches:
So the depth in inches vnto a fourth number:
which is the content of the base in inches.

As 144 hath vnto that fourth number:
So the length in feet to the content in feet.

And so in the same solid A H, the content will be found to be about 68 feet 62 parts.

As 1 vnto 21.6: so 30 vnto 648.
As 144 vnto 15.25: so 648 vnto 68.62.

Or as 144 vnto the bredth in inches:
So the depth in inches vnto a fourth number:
which is the content of the base in feet.

As 1 hath vnto that fourth number:
So the length in feet to the content in feet.

And so in the same solid AH, the content will be found to be about 68 feet 62 parts.

As 144 vnto 21.6: so 30 vnto 4.50:
As 1 vnto 4.50: so 15.25 vnto 68.62.

Or as 12 vnto the bredth in inches:
So the depth in inches vnto a fourth number.

As 12 vnto this fourth number:
So the length in feet to the content in feet.

And so also in the same solid AH, the content wil be found to be about 68 feet 62 parts.

As 12 vnto 21.6: so 30 vnto 54.
As 12 vnto 54: so 15.25 vnto 68.62.

All these varieties (and such like not here mentioned)

do

do follow vpon making of the base of the solid, to be E C; there would be as many more if any shall begin with the base E H, and so likewise if they make the base to be F D.

12 *Hauing the diameter of a cylinder giuen in inch measure, to find the length of a foot solid in inches.*

As the diameter in inches vnto 46.90:
So is 1 vnto a fourth number:
and that fourth to the length in inches.

So the diameter of a cylinder being 15 inches, the fourth number will be about 3.12, and the length of a foote solid 9 inches 78 parts.

As 15 vnto 46.90: so 1 vnto 3.127:
and so are 3.127 vnto 9.778.

13 *Hauing the diameter of a cylinder giuen in foot measure, to find the length of a foot solid in foot measure.*

As the diameter in feet vnto 1.128:
So is 1 vnto a fourth number;
and that fourth to the length in foot measure.

So the diameter being 1 foot 25 parts, the length of a foot solid wil be found about 8.14 parts of 1000.

As 1.25 vnto 1.128: so 1.00 to 0.9027:
and so are 9027 vnto 8148.

14 *Hauing the circumference of a cylinder giuen in inches, to find the length of a foot solid in inch measure.*

As the circumference in inches to 147.36:
So is 1 to a fourth number;
and that fourth to the length in inches.

So the circumference being 47 inches 13 parts, the length of a foot solid will be found about 9 inches 78 parts.

As 47.13 vnto 147.36: ſo 1.00 to 3.13:
and ſo are 3.13 vnto 9.78.

15 *Hauing the circumference of a cylinder giuen in foot measure, to find the length of a foot ſolid in foot measure.*

As the circumference in feete to 3.545:
So is 1 to a fourth number;
and that fourth to the length in foot meaſure.

So the circumference being 3 foot 927 parts, the length of a foot ſolid will be found to be about 815 parts.

As 3.927 vnto 3.545: ſo 1.000 vnto 0.903:
and ſo are 903 vnto 815.

16 *Hauing the ſide of a ſquare equall to the baſe of a cylinder, to find the length of a foot ſolid.*

The ſide of a ſquare equall to the circle, may be found by the eighth *Prop.* of broad meaſure, and then this *Prop.* may be wrought by the firſt and ſecond *Prop.* of ſolid meaſure.

17 *Hauing the diameter of a cylinder, and the length giuen in inches, to find the content in inches.*

As 1.128 vnto the diameter in inches:
So the length in inches to a fourth number;
and that fourth number to the content in inches.

So the diameter being 15 inches, and the length 105, the content of the cylinder will be found to be about 18560 inches.

As 1.1284 vnto 15: ſo are 105 vnto 1395.87:
and ſo are 1395.87 vnto 18555.34.

18 *Hauing the diameter and length of a cylinder in foot measure, to find the content in feet.*

As 1.128 to the diameter in feet:
So the length in feet to a fourth number;
and that fourth to the content in feet.

So the diameter being 1 foote 25 parts, and the length 8 foot and 75 parts, the content of the cylinder will be found about 10 foot 74 parts.

As 1.128 vnto 1.25: so 8.75 vnto 9.69:
and so are 9.69 vnto 10.737.

19 *Hauing the diameter of a cylinder, and the length giuen in inches, to find the content in feet.*

As 46.90 to the diameter in inches:
So the length in inches to a fourth number;
and that fourth to the content in feet.

So the diameter being 15 inches, and the length 105, the content will be found about 10 foot 74 parts.

As 46.906 vnto 15: so 105 vnto 33.58:
and so are 33.58 vnto 10.737.

20 *Hauing the diameter of a cylinder giuen in inches and the length in feet, to find the content in feet.*

As 13.54 to the diameter in inches:
So the length in feet to a fourth number;
and that fourth to the content in feete.

So the diameter being 15 inches, and the length 8 foote 75 parts, the content will be found about 10 foot 74 parts.

As 13.54 vnto 15: so 8.75 vnto 9.69;
and so are 9.69 vnto 10.74.

21 *Hauing the circumference and the length of a cylinder giuen in inches, to find the content in inches.*

As 3.545 to the circumference in inches:
So the length in inches to a fourth number;
and that fourth to the content in inches.

So the circumference being 47 inches 13 parts, and the length 105 inches, the content will be found about 18560 inches.

As 3.545 vnto 47.13: so 105 vnto 1396:
and so are 1396 vnto 18555.

22 *Hauing the circumference and length of a cylinder giuen in inches, to find the content in feet.*

As 147.36 to the circumference in inches:
So the length in inches to a fourth number;
and that fourth to the content in feet.

So the circumference being 47 inches 13 parts, and the length 105 inches, the content will be found about 10 foote 74 parts.

As 147.36 vnto 47.13: so 105 vnto 33.58:
and so are 33.58 vnto 10.74.

23 *Hauing the circumference and length of a cylinder giuen in foot measure, to find the content in feet.*

As 3.545 to the circumference in feet:
So the length in feet to a fourth number;
and that fourth to the content in feet.

So the circumference being 3 foote 927 parts, and the length 8 foot 75 parts, the content will be found to be 10 foot 74 parts.

As 3.545 vnto 3.927: so 8.75 vnto 9.69:
and so are 9.69 vnto 10.74.

24 ***Hauing the circumference of a cylinder giuen in inches, and the length in foot measure, to find the content in feete.***

As 42.54 to the circumference in inches:
So the length in feet to a fourth number;
and that fourth to the content in feet.

So the circumference being 47 inches 13 parts, and the length 8 foot 75 parts, the content will be found as before, 10 foot 74 parts.

As 42.54 vnto 47.13: so 8.75 vnto *9.69*:
and so are *9.69* vnto 10.74.

CHAP. IIII.

The vse of the line of Numbers in gaugeing of vessels.

THe vessels which are here measured, are supposed to be cylinders, or reduced vnto cylinders, by taking the mean betweene the diameter at the head and the diameter at the bongue, after the vsuall maner.

1 ***Hauing the diameter and the length of a vessell with the content thereof, to find the gauge point.***

Extend the compasses in the line of ***numbers*** to halfe the distance betweene the content and the length of the vessell, the same extent will reach from the diameter to the gauge point.

I put this proposition first, because these kind of measures are not alike in all places. Here at London it is said that a wine vessell being *66* inches in length, and 38 inches the diameter, would containe 324 gallons: which if it be true, we

may diuide the space betweene 324 and 66 into two equall parts, and the middle will fall about 146, and the same extent which reacheth from 324 to 146, wil reach from the diameter 38 vnto 17.15 the gauge point for a gallon of wine or oyle after London measure. The like reason holdeth for the like measures in all other places.

2 *Hauing the meane diameter and the length of a vessell, to find the content.*

Extend the compasses from the gauge point to the meane diameter, the same extent being doubled, shall giue the distance from the length to the content.

So the meane diameter of a wine vessell being 20 inches, and the length 25 inches, the content will be found to be 34 gallons after Londō measure. For extend the compasses from 17.15 vnto 20, the same extent wil reach from 25 vnto 29.15, and from 29.15 vnto 34.

In like maner if the meane diameter were 16 inches, and the length 23, the content would be found to be about 20 gallons. For the same extent which reacheth back from 17.15 vnto 16, will reach from 23 to 21.45, and from 21.45 vnto 20.

So that if the meane diameter shall be 17 inches and 15 centesmes or parts of 100, the number of inches in the length of the vessell, will giue the number of gallons contained in the same vessell: if the diameter shall be more or lesse then 17.15, the content in gallons will be accordingly more or lesse then the length in inches.

3 *Hauing the diameter and content, to find the length.*

Extend the compasses from the diameter to the gauge point, the same extent being doubled shall giue the distance from the content to the length of the vessell.

So the gauge point standing as before, if the diameter shal be 38 inches, and the content 324 gallons wine measure, the

length

length of the vessels will be found about **66** inches.

4 *Hauing the length of a vessell and the content, to find the diameter.*

Extend the compasses to halfe the distance betweene the length and the content, the same extent shall reach from the gauge point to the diameter.

So the length being **66** inches, and the content 3**24** gallons wine measure, the gauge point standing as before, the diameter of the vessell will be found to be about 38 inches.

CHAP. V.

Containing such Astronomicall propositions as are of ordinary vse in the practise of Nauigation.

1 *To find the altitude of the Sunne by the shadow of a gnomon set perpendicular to the horizon.*

As the parts of the shadow
  are to the parts of the gnomon:
So the tangent of 45 *gr.*
  to the tangent of the altitude.

Extend the compasses in the line of *numbers*, from the parts of the shadow to the parts of the *gnomon*; the same extent wil giue the distance from the tangent of 45 *gr.* to the tangent of the Sunnes altitude.

So the *gnomon* being 36, and the shadow **27**, the altitude will be found to be 36 *gr.* **52** *m.* Or the *gnomon* being **27**, and the shadow 36, the altitude will be found to be **53** *gr.* 8 *m.* Or the shadow being 20, and the gnomon 9, the altitude will be found to be **24** *gr.* **14** *m.* as in the eighth *Prop.* of the vse of the *tangent* line.

2 *Hauing the distance of the Sunne, from the next equinoctiall point, to find his declination.*

As the Radius is in proportion
to the sine of the Sunnes greatest declination:
So the sine of the Suns distance from the next equinoctiall point,
to the sine of the declination required.

Extend the compasses in the line of *sines*, from 90 *gr.* to 23 *gr.* 30 *m.* the same extent will giue the distance from the Sunnes place vnto his declination.

So the Sunne being either in 29 *gr.* of ♉, or 1 *gr.* of ♒, or 1 *gr.* of ♌, or 29 *gr.* of ♏, that is 59 *gr.* distant from the next equinoctiall point, the declination will be found about 20 *gr.*

If the Sunne be so neare the equinoctiall point that his declination fall to be vnder 1 *gr.* it may be found by the line of *numbers.* As if the Sunne were in 2 *gr.* 5 *m.* of ♈, that is, 125 *m.* from the equinoctiall point, the former extent of the compasses from the sine of 90 *gr.* to the sine of 23 *gr.* 30 *m.* will reach in the line of *numbers* from 125 vnto 50, which shewes the declination to be about 50 *m.*

3 *Hauing the latitude of the place, and the declination of the Sun, to find the time of the Suns rising and setting.*

As the cotangent of the latitude
to the tangent of the Suns declination:
So is the Radius
to the sine of the ascensionall difference betweene the houre of 6 and the time of the Suns rising or setting.

Extend the compasses from the tangent of the complement of the latitude, to the tangent of the declination: the same extent wil reach from the sine of 90 *gr.* to the sine of the ascensionall difference.

Or extend the compasses from the cotangent of the latitude

to

to the sine of 90 *gr*, the same extent will reach from the tangent of the declination to the sine of the ascensionall difference.

So the latitude being 51 *gr*. 30 *m*. Northward, and the declination 20 *gr*. the difference of ascension wil be found to be 27 *gr*. 14 *m*. which resolued into houres and minutes, doth giue 1 houre and almost 49 *m*. for the difference betweene the Sunnes rising or setting, and the houre of 6, according to the time of the yeare.

4 *Hauing the latitude of the place, and the distance of the Sun from the next equinoctiall point, to find his amplitude.*

As the cosine of the latitude
to the sine of the Suns greatest declination:
So the sine of the place of the Sun,
to the sine of the amplitude.

So the latitude being 51 *gr*. 30 *m*. and the place of the Sun in 1 *gr*. of ♒, that is 59 *gr*. distant from the next equinoctiall point, the amplitude will be found about 33 *gr*. 20 *m*. For extend the compasses in the line of *sines*, from 38 *gr*. 30 *m*. the sine of the complement of the latitude, vnto 23 *gr*. 30 *m*. the sine of the Suns greatest declination; the same extent will reach from 59 *gr*. vnto 33 *gr*. 20 *m*. Or extend them from 38 *gr*. 30 *m*. vnto 59 *gr*. the same extent will reach from 23 *gr*. 30 *m*. vnto 33 *gr*. 20 *m*. as before.

5 *Hauing the latitude of the place, and the declination of the Sun, to find his amplitude.*

As the cosine of the latitude
is to the Radius:
So the sine of the declination,
to the sine of the amplitude.

Extend the compasses from the cosine of the latitude to

the sine of 90 *gr.* the same extent will reach from the sine of the Suns declination to the sine of the amplitude.

Or extend them from the cosine of the latitude to the sine of the declination, the same extent will reach from the sine of 90 *gr.* to the sine of the amplitude.

So the latitude being 51 *gr.* 30 *m.* and the declination 20 *gr.* the amplitude will be found to be 33 *gr.* 20 *m.*

6 *Hauing the latitude of the place, and the declination of the Sun, to find the time when the Sun cometh to be due East or West.*

As the tangent of the latitude,
is to the tangent of the declination:
So the Radius
to the cosine of the houre from the meridian.

Extend the compasses from the tangent of the latitude to the tangent of the declination; the same extent will reach from the sine of 90 *gr.* to the sine of the complement of the houre.

Or extend them from the tangent of the latitude to the sine of 90 *gr*; the same extent will reach from the tangent of the declination to the sine of the complement of the houre.

So the latitude being 51 *gr.* 30 *m.* and the declination 20 *gr.* the Sunne will be 73 *gr.* 10 *m*: that is 4 houres and 53 *m.* frō the meridian, when he cometh to be in the East or West.

7 *Hauing the latitude of the place, and the declination of the Sun, to find what altitude the Sun shall haue, when he cometh to be due East or west.*

As the sine of the latitude
is to the sine of the declination:
So the Radius
to the sine of the altitude.

Extend the compasses in the line of *sines* from the latitude

to

to the ſine of the declination, the ſame extent will reach from the ſine of *90 gr.* to the ſine of the altitude.

Or extend them from the ſine of the latitude to the ſine of *90 gr*; the ſame extent will reach from the ſine of the declination to the ſine of the altitude.

So the latitude being *51 gr. 30 m.* and the declination 20 *gr.* the altitude will be found about *25 gr. 55 m.*

8 *Hauing the latitude of the place, and the declination of the Sun, to find what altitude the Sun ſhall haue at the houre of ſix.*

As the Radius is in proportion
  to the ſine of the ~~ſine of the~~ declination:
So the ſine of the latitude
  to the ſine of the altitude.

Extend the compaſſes in the line of *ſines*, from *90 gr.* to the declination; the ſame extent will reach from the latitude to the altitude.

Or extend them from *90 gr.* to the latitude, the ſame extent will hold from the declination to the altitude.

So the latitude being *51 gr. 30 m.* and the declination of the Sunne *20 gr.* the altitude of the Sun will be found to be about *15 gr. 30 m.*

9 *Hauing the latitude of the place, and the declination of the Sun, to find what azimuth the Sun ſhall haue at the houre of ſix.*

As the coſine of the latitude
  is to the Radius:
So the cotangent of the Suns declination,
  to the tangent of the azimuth from the North part
  of the meridian,

So

So the latitude being 51 *gr.* 30 *m.* and the declination 20 *gr.* the azimuth will be found to be 77 *gr.* 14 *m.* For extend the compasses in the line of *sines*, from 38 *gr.* 30 *m.* to 90 *gr.* the same extent will reach from the tangent of 70 *gr.* to the tangent of 77 *gr.* 14 *m.*

10 *Hauing the latitude of the place, and the declination of the Sun, and the altitude of the Sun, to find the azimuth.*

First consider the declination of the Sunne, whether it be toward the North or the South, so haue you his distance from your pole: then adde this distance, the complement of his altitude, and the complement of your latitude, all three together, and from halfe the summe subtract the distance from the pole, and note the difference.

1 As the Radius is in proportion
to the cosine of the altitude:
So the cosine of the latitude,
to a fourth sine.

2 As this fourth sine
is to the sine of the halfe summe:
So the sine of the difference,
to a seuenth sine.

Then find a meane proportionall betweene this seuenth sine and the Radius, this meane shall be the sine of the complement of halfe the azimuth from the North part of the meridian.

Suppose the declination of the Sun being knowne by the time of the yeare to be 20 *gr.* Southward, the altitude aboue the horizon found by obseruation 12 *gr.* and the latitude Northwards 51 *gr.* 30 *m.* it were required to find the azimuth. The declination is Southward, and therefore the distance from the pole 110 *gr*; then turning the altitude and latitude vnto their complements, I adde them all three together, and from halfe the summe subtract the distance from the pole, noting

noting the difference after this maner.

Declin. South	20 gr.	0 m.	The distance	110 gr.	0 m.
Altitude	12	0	The complement	78	0
Latitude N.	51	30	The complement	38	30
			The summe of all three	226	30
			The halfe summe	113	15
			The difference	3	15

This done, I come to the Staffe, and extend the compasses from the sine of 90 *gr.* to the sine of 78 *gr.* and find the same extent to reach from the sine of 38 *gr.* 30 *m.* vnto 37 *gr.* 30 *m.* Or if I extend them from 90 *gr.* to 38 *gr.* 30 *m.* the same extent doth reach from 78 *gr.* vnto 37 *gr.* 30 *m.* which is the fourth sine required.

Then I extend the compasses againe, from this fourth sine of 37 *gr.* 30 *m.* vnto the sine of the halfe summe 113 *gr.* 15 *m.* that is to the sine of 66 *gr.* 45 *m.* (for after 90 *gr.* the sine of 80 *gr.* doth stand for a sine of 100 *gr.* and the sine of 70 *gr.* for a sine of 110 *gr.* and so the rest for those which are their complements to 180 *gr.*) and this second extent doth reach from the sine of the difference 3 *gr.* 15 *m.* to the sine of 4 *gr.* 54 *m.* Or if I extend them from the fourth sine of 37 *gr.* 30 *m.* to the sine of the difference 3 *gr.* 15 *m.* the same extent will reach from the sine of the halfe summe 113 *gr.* 15 *m.* vnto 4 *gr.* 54 *m.* which is the seuenth sine required.

Lastly, I diuide the space betweene this seuenth sine of 4 *gr.* 54 *m.* and the sine of 90 *gr.* into two equall parts, and I find the meane proportionall sine to fall on 17 *gr.* whose complement is 73 *gr*; the double of 73 *gr.* is 146 *gr.* and such is the azimuth required.

Or hauing found the seuenth sine to be 4 *gr.* 54 *m.* I might looke ouer against it, in the line of *versed sines*, and there I should find 146 *gr.* for the azimuth from the North part of the meridian; and the complement of 146 *gr.* to a semicircle being 34 *gr.* will giue the azimuth from the South part of the meridian.

But if it were required to find the azimuth in the same latitude of 51 *gr.* 30 *m.* Northward, with the same altitude of

12 *gr.* and like declination of 20 *gr.* to the Northward, it would be found to be onely 72 *gr.* 52 *m.* though the maner of worke be the same as before.

Declin. North	20 *gr.* 0 *m*	The distance is	70 *gr.* 0 *m.*		
Altitude	12	0	The complement	78	0
Latitud. North	51	30	The complement	38	30
		The summe of all three		186	30
		The halfe summe		93	15
		The difference		23	15

Here as the Radius is to the sine of 78 *gr*: so the sine of 38 *gr.* 30 *m.* to the sine of 37 *gr.* 30 *m.* which is the fourth sine, and the same as before.

Then as this fourth sine of 37 *gr.* 30 *m.* is to the sine of 93 *gr.* 15 *m*: so the sine of 23 *gr.* 15 *m.* to the sine of 40 *gr.* 20 *m.* which is the seuenth sine.

The halfe way betweene this seuenth sine and the sine of 90 *gr.* doth fall at 53 *gr.* 34 *m.* whose complement is 36 *gr.* 26 *m*; and the double of that is 72 *gr.* 52 *m.* the azimuth required.

Or I may find this same azimuth in the line of *versed sines*, ouer against the seuenth sine of 40 *gr.* 20 *m.*

11 *Hauing the latitude of the place, the declination of the Sun, and the altitude of the Sun, to find the houre of the day.*

Adde the complement of the Suns altitude, and the distance of the Sun from the pole, and the complement of your latitude, all three together, and from halfe the summe subtract the complement of the altitude, and note the difference.

1 As the Radius is in proportion
  to the sine of the Suns distance from the pole:
So the sine of the complement of the latitude,
  to a fourth sine.

2 As

2 As this fourth sine
is to the sine of the halfe summe:
So the sine of the difference
to a seuenth sine.

The meane proportionall betweene this seuenth sine and the sine of 90 *gr.* will be the sine of the complement of halfe the houre from the meridian.

Thus in our latitude of 51 *gr.* 30 *m.* the declination of the Sun being 20 *gr.* Northward, and the altitude 12 *gr.* I might find the Sun to be 95 *gr.* 52 *m.* from the meridian.

Altitude	12 *gr.*	0 *m.*	The complement is	78 *gr.*	0 *m.*
Declin. North	20	0	the dist. from the pole	70	0
Latitude	51	30	the complement is	38	30
			The summe of all three	186	30
			The halfe summe	93	15
			The difference	15	15

Here as the Radius is to the sine of 70 *gr.*
So the sine of 38 *gr.* 30 *m.* to the sine of 35 *gr.* 48 *m.*
As this sine of 35 *gr.* 48 *m.* is to the sine of 93 *gr.* 15 *m.*
So the sine of 15 *gr.* 15 *m.* to the sine of 26 *gr.* 40 *m.*

The halfe way betweeen this seuenth sine of 26 *gr.* 40 *m.* and the sine of 90 *gr.* doth fall at 42 *gr.* 4 *m.* whose complement is 47 *gr.* 56 *m.* and the double of that, 95 *gr.* 52 *m.* which conuerted into houres, doth giue 6 houres and almost 24 *m.* from the meridian.

Or I might find these 95 *gr* 52 *m.* in the line of *versed sines*, ouer against the seuenth sine of 26 *gr.* 40 *m.*

12 *Hauing the azimuth, the Suns altitude, and the declination, to find the houre of the day.*

As the cosine of the declination
is to the sine of the azimuth:
So the cosine of the altitude
to the sine of the houre.

Thus the declination being 20 *gr.* Southward, the altitude 12 *gr.* and the azimuth found by the tenth *Prop.* 146 *gr.* I might find the time to be 35 *gr.* 36 *m.* that is 2 houres 21 *m.* from the meridian.

13 *Hauing the houre of the day, the Sunnes altitude, and the declination, to find the azimuth.*

As the cosine of the altitude
is to the sine of the houre:
So the cosine of the declination,
to the sine of the azimuth.

So the altitude of the Sun being 12 *gr.* and the declination 20 *gr.* Southward, and the angle of the houre 35 *gr.* 36 *m.* I should find the azimuth to be 34 *gr.* And so it is if it be reckoned from the South; but 146 *gr.* if it be taken from the North part of the meridian.

14 *Hauing the distance of the Sun from the next equinoctiall point, to find his right ascension.*

As the Radius
to the cosine of the greatest declination:
So the tangent of the distance,
to the tangent of the right ascension.

So the Sun being in the first degree of ♒, that is 59 *gr.* distant from the next equinoctiall point, and the greatest declination 23 *gr.* 30 *m.* the right ascension will be found to be 56 *gr.* 46 *m.* short of the beginning of ♈; and therefore 303 *gr.* 14 *m.*

15 *Hauing the declination of the Sun, to find his right ascension.*

As the tangent of the greatest declination
is to the tangent of the declination giuen:

So the Radius
to the ſine of the right aſcenſion.

So the greateſt declination being 23 *gr.* 30 *m.* and the declination of the Sun giuen 20 *gr.* the right aſcenſion will be found about 56 *gr.* 50 *m.*

Theſe are ſuch Aſtronomicall propoſitions as I take to be vſefull for Sea-men. For the firſt and ſecond will help them to find their latitude; the third to find the Suns riſing and ſetting; the 4.5.6.7.8.9.10.13. *Prop.* to finde the variation of their compaſſe; the 11 and 12 *Prop.* to find the houre of the day; and the two laſt toward the finding of the houre of the night. For hauing the latitude of the place, with the declination and altitude of any ſtarre, they may find the houre of the ſtarre from the meridian, as in the 11 *Prop.* Then comparing the right aſcenſion of the ſtarre with the right aſcenſion of the Sunne, they may haue the houre of the night.

All theſe propoſitions and ſuch others may be wrought alſo by the tables of *ſines* and *tangents*. For where foure numbers do hold in proportion; as the firſt to the ſecond, ſo the third to the fourth; there if we multiply the ſecond into the third, and diuide the product by the firſt, the quotient will giue the fourth required. As in the example of the laſt *Prop.* where the delination being giuen, it was required to find the right aſcenſion. The tangent of 20 *gr.* the declination giuen is 3639702, which being multiplied by the Radius, the product is 36397020000000, and this diuided by 4348124 the tangent of 23 *gr.* 30 *m.* the quotient is 8370741 the ſine of 56 *gr.* 50 *m.* for the right aſcenſion required.

Or if any will vſe my tables of *artificiall ſines* and *tangents*, they may adde the ſecond and the third together, and from the ſumme ſubtract the firſt, the remainder will giue the fourth required. And ſo my tangent of 20 *gr.* is 9561.0658, which being added to the Radius, makes 19561.0658; from this if they ſubtract 9638.3019 the tangent of 23 *gr.* 30 *m.* they ſhall find the remainder to be 9922.7639, which in my

Canon is the sine of 56 *gr*. 49 *m*. 56 *secōds*; and such is the right ascension required, if it be reckoned from the next equinoctiall point.

The like reason holdeth for all other Astronomicall propositions, as I will farther shew by those two examples which I gaue before for the finding of the azimuth in the 10 *Prop.* because they are thought to be harder then the rest, and require three operations.

In the first example.

Declin. South	20 *gr*.	0 *m*.	The distance	110 *gr*.	0 *m*.
Altitude	12	0	the complement	78	0
Latitude Nor.	51	30	the complement	38	30
			The summe of all three	226	30
			The halfe summe	113	15
			The difference	3	15

The first operation will be to finde the fourth sine; and that is done by adding the sine of the complement of the altitude to the sine of the complement of the latitude, and subtracting the Radius: so adding 9990.4044 the sine of 78 *gr*. vnto 9794.1495 the sine of 38 *gr*. 30 *m*. the summe wll be 19784.5539. And the Radius being subtracted, the remainder 9784.5539 is the fourth sine, and belongeth to 37 *gr*. 30 *m*.

The second operation will be to find the seuenth sine; and that is done by adding the sine of the halfe summe to the sine of the difference, and subtracting the fourth sine. So the halfe summe being 113 *gr*. 15 *m*. I take his complement to a semicircle, and so find his sine to be 9963.2168, to which I adde 8753.5278, the sine of the difference 3 *gr*. 15 *m*; and the summe is 18716.7446. From this I take the fourth sine 9784.5539, and the remainder will be 8932.1907, which is the seuenth sine, and belongeth to 4 *gr*. 54 *m*.

The third operation will be to finde the meane proportionall sine betweene the seuenth sine and the Radius. This in common arithmetique is done by multiplying the two extremes, and taking the square roote of the product. As in

finding

finding a meane proportionall betweene 4 and 9, we multiply 4 into 9, and the product is 36, whose square root is 6, the meane proportionall between 4 and 9. But here it is done by adding the sine and the Radius, and taking the halfe of them. So the summe of the last seuenth sine and the Radius is 18932.1907 and the halfe of that 9466.0953, which is the meane proportionall sine required, and belongeth to 17 *gr.* whose complement is 73 *gr.* and the double of that 146 *gr.* the same azimuth as before.

In the second example.

Declin.North	20 *gr.* 0 *m.*	The distance	70 *gr.*	0 *m.*
Altitude	12 0	the complement	78	0
Latitud.North	51 30	the complement	38	30
		The summe of all three	186	30
		The halfe summe	93	15
		The difference	23	15

The first operation will be to finde the fourth sine; and that is here 9784.5539, as in the former example.

The second operation wil be to find the seuenth sine; and so here the sine of the halfe summe 93 *gr.* 15 *m.* being the same with the sine of 86 *gr.* 45 *m.* his complement to 180 *gr.* I find it to be 9999.3009, to which I adde 9596.3153 the sine of the difference 23 *gr.* 15 *m.* and the summe is 19595.6162. From this I take the fourth sine 9784.5539, and the remainder will be 9811.0623 for the seuenth sine, and belongeth to 40 *gr.* 20 *m.*

The third operation will be to find the meane proportionall sine betweene the seuenth sine and the Radius. And so here the Radius being added to the seuenth sine, the summe will be 19811.0623, and the halfe of that 9905.5311 doth giue the meane proportionall sine belonging to about 53 *gr.* 34 *m.* whose complement is 36 *gr.* 26 *m.* & the double of that 72 *gr.* 52 *m.* the same azimuth as before.

I haue set downe these three examples thus particularly, that I might shew the agreement between the *Staffe* and the *Canon.* But otherwise I might deliuer both the precept and

the

the worke, for the two last, more compendiously. For generally in all sphericall triangles, where three sides are knowne, and an angle required, make that side which is opposite to the angle required, to be the base; and gather the summe, the halfe summe, and the difference as before.

As the rectangle contained vnder the sines of the sides,
is to the square of the whole sine:
So the rectangle contained vnder the sines of the halfe
summe and the difference,
to the square of the cosine of halfe the angle.

Then for the worke, we may for the most part leaue out the two last figures; and if they be aboue 50, put an vnitie to the sixt place, after this maner.

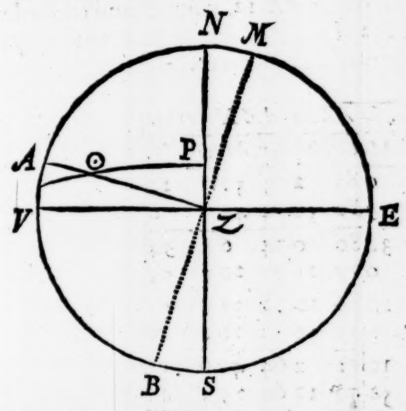
The second example.

70 *gr.*	0 *m*				
78	0	9990	40		
38	30	9794	15		
186	30	19784	55		
93	15	9999	30		
23	15	9596	32		
		20000	00		
		39595	62		
		19811	07		
36	26	9905	53	53 *gr.*	34 *m.*
72	52			107	8

Or for such numbers as are to be subtracted, I may take them out of the Radius, and write downe the residue, and then adde them together with the rest. As in the same second example, the sines of 78 *gr.* and of 38 *gr.* 30 *m.* being the numbers to be subtracted; if I take 9990.4044 the sine of 78 *gr.* out of the Radius 10000 0000, the residue is 9.5956: and so the residue of 9794.1495 is 205.8505. Wherefore in stead of subtracting those sines, I may adde these residues after this maner:

70 *gr.* 0 *m.*			
78	0	9'59	
38	30	205 85	
186	30		
93	15	9999 | 30	
23	15	9596 | 32	
		19811 06	
36	26	9905,53	53 *gr.* 34 *m.*
72	52		107 8

Hauing these meanes to find the Sunnes azimuth, we may compare it with the magneticall azimuth, and so finde the variation of the needle.



For let the circle *A M B*, drawne on the center *Z*, be a plane, parallell to the horizon; *A* the point whereon the Sun beareth from vs, *M* the North point of the magneticall needle, and the angle *A Z M* the magneticall azimuth. If we find the Sunnes azimuth as before, to be *72 gr. 52 m.* from the North to the Westward, we may allow so many degrees from *A* vnto *N*, and so we haue the true North point of the meridian, and consequently the East, South, and West points of the horizon; and the distance betweene *N* and *M* shall be

the

the variation of the needle. So that if the magneticall azimuth *AZM* shall be 84 *gr.* 7 *m.* and the Suns azimuth *AZN* 72 *gr.* 52 *m.* then must *NZM* the difference betweene the two meridians, giue the variation to be 11 *gr.* 15 *m.* as M^r. *Bourough* heretofore found it by his obseruations at *Limehouse* in the yeare 1580. But if the magneticall azimuth *AZM* shall be 79 *gr.* 7 *m.* and the Suns azimuth *AZN* 72 *gr.* 52 *m.* then shall the variation *NZM* be only 6 *gr.* 15 *m.* as I haue sometimes found it of late. Herevpon I enquired after the place where M^r. *Bourough* obserued, and went to *Limehouse* with some of my friends, and tooke with vs a quadrant of 3 foote semidiameter, and two needles, the one aboue 6 inches, and the other 10 inches long, where I made the semidiameter of my horizontall plane *AZ* 12 inches: and toward night the 13 of Iune 1622, I made obseruation in seuerall parts of the ground, and found as followeth.

Alt. ☉		*AZM*		*AZN*		*Variat*	
Gr.	*M.*	*Gr.*	*M.*	*Gr.*	*M.*	*Gr.*	*M.*
19	0	82	2	75	52	6	10
18	5	80	50	74	44	6	6
17	34	80	0	74	6	5	54
17	0	79	15	73	20	5	55
16	18	78	12	72	32	5	40
16	0	77	50	72	10	5	40
10	10	71	2	64	49	6	13
9	52	70	12	64	25	5	47

CHAP.

CHAP. VI.

Containing ſuch nauticall queſtions, as are of ordinary vſe, concerning longitude, latitude, Rumb, and diſtance.

1 *To keep an account of the ſhips way.*

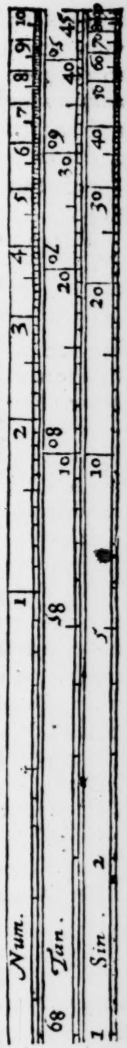
THe way that the ſhip maketh, may be knowne to an old ſea-man by experience, by others it may be found for ſome ſmall portion of time, either by the log line, or by the diſtance of two knowne markes on the ſhips ſide. The time in which it maketh this way, may be meaſured by a watch, or by a glaſſe. Then as long as the wind continueth at the ſame ſtay, it followeth by proportion,

As the time giuen is to an houre:
So the way made, to an houres way.

Suppoſe the time to be 15 ſeconds, which make a quarter of a minute, and the way of the ſhip 88 feet: then becauſe there are 3600 ſeconds in an houre, I may extend the compaſſes in the line of *numbers*, from 15 vnto 3600, and the ſame extent will reach from 88 vnto 21120.

Or I may extend them from 15 vnto 88, and this extent will reach from 3600 vnto 21120; which ſhewes that an houres way came to 21120 feete.

But this were an vnneceſſary buſineſſe, to hearken after feet or fadoms. It ſufficeth our ſea-men to find the way of their ſhip in leagues or miles. And they ſay that there are 5 feet in a pace, 1000 paces in a mile, and 60 miles in a degree, and therefore 300000 feete in a degree, Yet compa-



ring seuerall obseruations, and their measures with our feete vsuall about *London*, I find that we may allow 352000 feete to a degree; and then if I extend the compasses in the line of *numbers* from 352000 vnto 21120, I shall find the same extent to reach from 20 leagues the measure of one degree, to 1.2, and from 60 miles to 3.6; which shewes the houres way to be 1 league and 2 tenths of a league, or 3 miles and 6 tenths of a mile.

But to auoid these fractions and other tedious reductions, I suppose it would be more easie to keep this account of the ships way (as also of the difference of latitude, and the difference of longitude) by degrees and parts of degrees, allowing 100 parts to each degree, which we may therefore call by the name of *centesmes*. Neither would this be hard to conceiue. For if 100 such parts do make a degree, then shall 50 parts be equall to 30 minutes, as 30 minutes are equall to 10 leagues. And 5 parts shall be equall to 3 minutes, as 3 minutes are equall to 1 league. And so the same extent as before, will reach from 100 parts vnto 6; which shewes that the houres way required is 6 *cent.* such as 100 do make a degree, and 5 do make an ordinary league.

This might also be done at one operation. For vpon these suppositions, diuide 44 feet into 45 lengths, and set as many of them as you may conueniently betweene two markes on the ships side, and note the seconds of time in which the ship goeth these lengths: so the lengths diuided by the time, shall giue the *cent.* which the ship goeth in an houre.

Suppose the distance betweene the two markes to be 60 lengths (which are 58 feet and 8 inches) and let the time be 12 seconds: extend the compasses from 12 to 1, in the line of *numbers*; so the same extent will reach from 60 vnto 5. Or extend them from 12 vnto 60, and the same extent will reach from 1 vnto 5. This shewes that the ships way is according to 5 *cent.* in an houre.

This may be found yet more easily, if the log line shall be fitted to the time. As if the time be 45 seconds, the log line may haue a knot at the end of euery 44 feete; then doth the

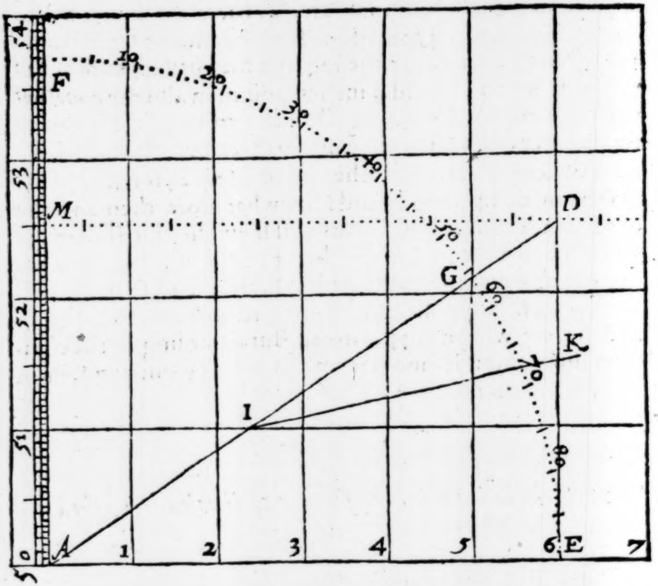
ship

ſhip run ſo many *cent.* in an houre, as there are knots vered out in the ſpace of 45 ſeconds. If 30 ſeconds do ſeeme to be a more conuenient time, the log line may haue a knot at the end of euery 29 feet and 4 inches; and then alſo the *centeſmes* will be as many as the knots. Or if the knots be made to any ſet number of feet, the time may be fitted vnto the diſtance. As if the knots be made at the end of euery 24 feet, the glaſſe may be made 24 ſeconds and ſomewhat more then an halfe of a ſecond; and ſo theſe knots will ſhew the *cent.* If there be 5 knots vered out in a glaſſe, then 5 *cent*; if 6 knots, then the ſhip goeth 6 *cent.* in the ſpace of an houre; and ſo in the reſt. For vpon this ſuppoſition, the proportion between the time and the feet will be as 45 vnto 44. But according to the common ſuppoſition it ſhould ſeeme to be as 45 vnto 37 $\frac{1}{2}$, or in leſſer termes as 6 vnto 5. Thoſe which are vpon the place, may make proofe of both, and follow that which agrees beſt with their experience.

2 *By the latitude and difference of longitude, to find the diſtance vpon a courſe of Eaſt and Weſt.*

Extend the compaſſes from the ſine of 90 *gr.* vnto the ſine of the complement of the latitude; the ſame extent ſhal reach in the line of *numbers* from the difference of longitude to the diſtance.

So the meaſure of one degree in the equator, being 100 *cent.* the diſtance belonging to one degree of longitude in the latitude of 51 *gr.* 30 *m.* will be found about 62 *cent.* and $\frac{1}{4}$. Or if the meaſure of a degree be 60 miles, the diſtance will be found about 37 miles and $\frac{1}{3}$. If the meaſure be 20 leagues, then almoſt 12 leagues and $\frac{1}{2}$. If the meaſure be 17 $\frac{1}{2}$, as in the Spaniſh charts, then ſomewhat leſſe then 11 leagues ſailing vpon this parallell, will giue an alteration of one degree of longitude.



3 *By the latitude and distance vpon a course of East or west, to find the difference of longitude.*

Extend the compasses from the sine of the complement of the latitude, to the sine of 90 *gr*; the same extent wil reach in the line of *numbers* from the distance to the difference of longitude.

So the distance vpon a course of East or West, in the latitude of 51 *gr*.30 *m*. being 100 *cent*. the difference of longitude will be found 1.60, which make one degree and 60 *centesmes* or 1 *gr*.36 *m*.

Or if it be 60 miles, the difference of longitude wil be 96, which also make 1 *gr*.36 *m*. as before.

4 *The*

4 *The longitude and latitude of two places being giuen, to find the Rumb leading from the one to the other.*

Extend the compasses in the line of *numbers* from the difference of latitudes to the difference of longitudes; the same extent will giue the distance from the tangent of 45 *gr.* vnto the tangent of the Rumb, according to the proiection of the common sea-chart.

So the latitude of the first place being 50 *gr.* the latitude of the second 52 *gr.* 30 *m.* and the difference of longitude 6 *gr.* the Rumb will be found to be about 67 *gr.* 23 *m.* which is neare the inclination of the sixth Rumb to the meridian. But this Rumb so found, is alwayes greater then it should be, and therefore to be limited; which may be done sufficiently for the Sea-mans vse, after this maner:

Extend the compasses either from the sine of 90 *gr.* vnto the sine of the complement of the middle latitude, the same extent will reach frõ the tangent of the Rumb before found, to the tangent of the Rumb limited.

Or else extend them from the sine of 90 *gr.* vnto the tangent of the Rumb before found; the same extent will reach from the sine of the complement of the middle latitude, vnto the tangent of the Rumb limited.

So the middle latitude between 50 *gr.* and 52 *gr.* 30 *m.* being 51 *gr.* 15 *m.* and the Rumb before found 67 *gr.* 23 *m.* the Rumb limited will be found to be about 56 *gr.* 20 *m.* which is but fiue minutes more then the inclination of the fift Rumb to the meridian.

2 This Rumb may be found by the help of the *meridian line* vpon the Staffe. For if I take the difference of latitude out of the *meridian line* from 50 *gr.* vnto 52 *gr.* 30 *m.* and measure it in his equinoctiall, or at the beginning of the *meridian line*, I shall find it there to be equal to 4 *gr.* Wherefore I work as if the difference of latitude were 4 *gr.* and extend the compasses in the line of *numbers* from 4 vnto 6: so shall I finde

the

the same extent to reach from the tangent of 45 *gr.* vnto the tangent of 56 *gr.* 20 *m.* and this is the inclination of the Rumb required.

5 *By the Rumb and both latitudes, to find the distance vpon the Rumb.*

Extend the compasses from the sine of the complement of the Rumb, vnto the sine of 90 *gr.* the same extent in the line of *numbers* shall reach from the difference of latitude vnto the distance vpon the Rumb.

So the latitude of the first place being 50 *gr.* the latitude of the second 52 *gr.* 30 *m.* and the Rumb the fift from the meridian. If I extend the compasses from 33 *gr.* 45 *m.* vnto the sine of 90 *gr.* I shall find the same extent in the line of *numbers* to reach from 2. *gr.* 50 *cent.* to 4 *gr.* 50 *cent.* and such is the distance required.

6 *By the distance and both latitudes to find the Rumb.*

Extend the compasses in the line of *numbers* from the distance vnto the difference of latitudes; the same extent will reach in the line of *sines*, from 90 *gr.* vnto the complement of the Rumb.

So the one place being in the latitude of 50 *gr.* the other in the latitude of 52 *gr.* 30 *m.* and the distance between them 4 *gr.* 50 *cent.* If I extend the compasses from 4.50 vnto 2.50 in the line of *numbers*, I shall find the same extent to reach from the sine of 90 *gr.* vnto the complement of 56 *gr.* 15 *m.* and such is the inclination of the Rumb required.

7 *By one latitude, Rumb, and distance, to find the difference of latitudes.*

Extend the compasses in the line of *sines*, from 90 *gr.* vnto the complement of the Rumb; the same extent in the line of

numbers

numbers, will reach from the diſtance, vnto the difference of latitudes.

So the leſſer altitude being 50 *gr.* and the diſtance 4 *gr.* 50 *cent.* vpon the fifth Rumb from the meridian: if I extend the compaſſes from the ſine of 90 *gr.* to 33 *gr.* 45 *m.* I ſhall finde the ſame extent to reach from 4.50 in the line of *numbers*, vnto 2.50; and therefore the ſecond latitude to be 52 *gr.* 30 *m.*

8 *By the Rumb and both latitudes, to find the difference of longitude.*

Extend the compaſſes from the tangent of 45 *gr.* vnto the tangent of the Rumb; the ſame extent will reach in the line of *numbers* from the difference of latitudes vnto the difference of longitude, according to the proiection of the common ſea-chart.

So the firſt latitude being 50 *gr.* and the ſecond 52 *gr.* 30 *m.* and the Rumb the fifth from the meridian: if I extend the compaſſes from the tangent of 45 *gr.* vnto 56 *gr.* 15 *m.* I ſhall find the ſame extent to reach from 2.50 in the line of *numbers* to about 3.75, which make 3 *gr.* 45 *m.* But this difference of longitude ſo found, is alwayes leſſer then it ſhould be, and therefore to be enlarged, which may be done ſufficiently for the ſea-mens vſe, after this maner:

Extend the compaſſes from the ſine of the complement of the middle latitude, vnto the ſine of 90 *gr.* the ſame extent will reach in the line of *numbers* from the difference of longitude before found, vnto the difference of longitude inlarged.

So the middle latitude in this example being 51 *gr.* 15 *m.* and the difference of longitude before found 3 *gr.* 75 *cent.* the difference of longitude inlarged will be found about 5 *gr.* 99 *cent.* which are neare 6 *gr.*

2 This difference of longitude may be found by help of the *meridian line* vpon the Staffe. For if I take the proper difference of latitude out of the meridian line, and meaſure it in his equinoctiall, or at the beginning of the meridian line,

I ſhall find it to be equall to foure of thoſe degrees. Wherefore hauing extended the compaſſes as before from the tangent of 45 *gr.* vnto the tangent of 56 *gr.* 15 *m.* the ſame extent will reach from 4.00 in the line of *numbers,* vnto 5.99: which ſhewes the difference of longitude to be about 5 *gr.* 99 *cent.* or about halfe a minute ſhort of ſix degrees.

9 *By the Rumb and both latitudes, to finde the diſtance belonging to the chart of* Mercators *proiection.*

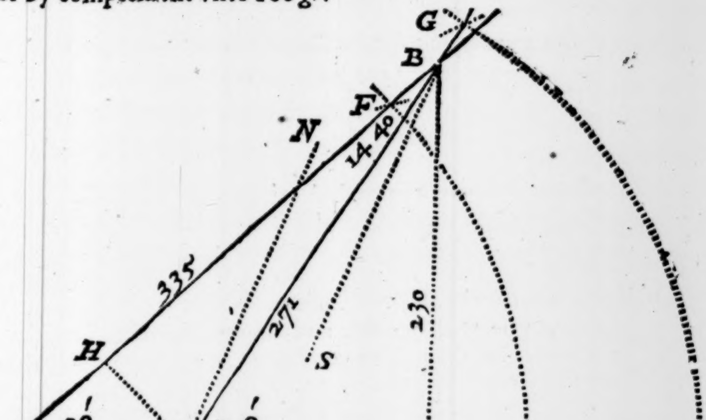
Take the proper difference of latitudes out of the meridian line of the chart, and meaſure it in his equinoctiall, or one of the parallels, and it will there giue the difference of latitudes inlarged. Then extend the compaſſes from the ſine of the complement of the Rumb vnto the ſine of 90 *gr.* the ſame extent will reach in the line of *numbers,* from the latitude inlarged, vnto the diſtance required. Or extend them from the complement of the Rumb to the latitude inlarged, the ſame extent will reach from 90 *gr.* vnto the diſtance.

For example, let the place giuen be *A* in the latitude of 50 *gr.* *D* in the latitude of 52 *gr.* 30 *m.* *A M* the difference of latitudes, and the Rumb *M A D* the fifth from the meridian. Firſt I take out *A M* the difference of latitudes, and meaſure it in *A E* one of the parallels of the equinoctiall; I find it to be very neare 4 *gr*: this is the difference of latitudes inlarged. Then if I extend the compaſſes from the ſine of 33 *gr.* 45 *m.* the complement of the fifth Rumb vnto the ſine 90 *gr.* I ſhall find the ſame extent to reach in the line of *numbers* from 4.00 vnto 7.20. And this is the diſtance belonging to the chart. Wherefore I take out theſe 7 *gr.* 20 *cent.* out of the ſcale of the parallell *A E,* and pricke it downe vpon the Rumb from *A* vnto *D,* where it meeteth with the parallell of the ſecond latitude. Laſtly, I meaſure it in the *meridian line,* ſetting one foote of the compaſſes as much below the leſſer latitude as the other aboue the greater latitude, and find it to be 4 *gr.* 50 *cent.* which is the ſame diſtance that I found before in the 5. *Prop.*

10. By

10 *By the way of the ſhip, and two angles of poſition, to find the diſtance betweene the ſhip and the land.*

The way of the ſhip may be knowne as in the firſt *Prop.* The angles may be obſerued either by the Staffe, or by a needle ſet on the Staffe. For example, ſuppoſe that being at *A*, I had ſight of the land at *B*, the ſhip going Eaſt Northeaſt from *A* toward *C*, and the angle of the ſhips poſition *BAC* being 43 *gr.* 20 *m*: and after that the ſhip had made 10 *cent.* or 2 leagues of way from *A* vnto *D*, I obſerued againe, and found the ſecond angle of the ſhips poſition *BDC* to be 58 *gr.* or the inward angle *BDA* to be 122 *gr.* then may I finde the third angle *ABD* to be 14 *gr.* 40 *m.* either by ſubtraction or by complement vnto 180 *gr.*



In this and the like caſes, I haue a right line triangle, in which there is one ſide and three angles knowne, and it is required to finde the other two ſides and the *Canon* for it, is this:

As the sine of the angle opposite to the knowne side,
is to that knowne side:
So the sine of the angle opposite to the side required,
is to the side required.

Wherefore I extend the compasses from 14 *gr.* 40 *m.* in the *sines*, to 10 in line of *numbers*, and this extent doth reach from 58 *gr.* to 33 $\frac{1}{2}$, and such is the distance between *A* and *B*, and it reacheth from 43 *gr.* 20 *m.* vnto 27 in the line of *numbers*; and such is the distance from *D* to *B*.

These two distances being knowne, I may set out the land vpon the chart. For hauing set downe the way of the ship from *A* to *D* by that which I shewed before in the vse of the *meridian line*, I may by the same reason set off the distance *A B* and *D B*, which meeting in the point *B*, shall there resemble the land required.

11 *By knowing the distance between two places on the land, and how they beare one from the other, and hauing the angles of position at the ship to find the distance betweene the ship and the land.*

If it may be conueniently, let the angle of position be obserued at such time as the ship cometh to be right ouer against one of the places. As if the places be East and West, seeke to bring one of them South or North from you, and then obserue the angle of position: so shall you haue a right line triangle, with one side and three angles, whereby to find the two other sides. First you haue the angle of position at the ship; then a right angle at the place that is ouer against you; and the third angle at the other place is the complement to the angle of position. Wherefore

As the sine of the angle of position,
is to the distance betweene the two places:
So the cosine of the angle of position,
to the distance betweene the ship and the nearer place.

And

And so is the sine of 90 *gr.*
to the distance from the ship to the farther place.

So the places being 15 *cent.* or three leagues one from the other, and the angle of position 29 *gr*; the nearer distance will be found about 27 *cent.* and the farther distance about 31 *cent.*

Or howsoeuer the angle of position were obserued, the distance betweene the ship and the land may be found generally as in this example:

Suppose *A* and *D* were two head lands knowne to be East Northeast, and West Southwest, 10 *cent.* or two leagues one from the other; and that the ship being at *B*, I obserued the angle of the ships position *DBA*, and found it to be 14 *gr.* 40 *m.* and that D did beare 9 *gr.* 30 *m.* and *A* 24 *gr.* 10 *m.* from the meridian *BS*; this example would be like the former. For if the angle *SBD* be 9 *gr.* 30 *m.* from the South to the Westward, then shall *NDB* be 9 *gr.* 30 *m.* from the North to the Eastward. Take these 9 *gr.* 30 *m.* out of the angle *NDE* which is 67 *gr.* 30 *m.* because the two head lands lie East Northeast, and there will remaine 58 *gr.* for the angle *BDE*, and the inward angle BDA shall be 122 *gr.* Take these two angles A B D and B D A out of 180 *gr.* and there wil remaine 43 *gr* 20 *m* for the third angle BAD. Wherefore here also are three angles and one side, by which I may find the two other sides, as in the last *Prop.*

These propositions thus wrought by the Staffe, are such as I thought to be vsefull for sea-men, and those that are skilfull may apply the example to many others. Those that begin, and are willing to practise, may busie themselues with this which followeth.

Suppose foure ports, *L, N, O, P*; of which *L* is in the latitude of 50 *gr.* *N* is North from *L* 200 leagues or 1000 *centesmes*; *O* West from *L* 1000 *cent.* and *P* West from *N* 1000 *cent*: so that *L* and *O* will be in the same latitude of 50 *gr.* *N* and *P* both in the latitude of 60 *gr.* Then let two ships depart from *L*, the one to touch at *O*, the other at *N*,

and then both to meet at *P*, there to lade, and from thence to returne the nearest way vnto *L*. Here many questions may be proposed.

1 What is the longitude of the port at *O*?

2 What is the longitude of *P*? And why *O* and *P* should not be in the same longitude?

3 What is the Rumb from *O* vnto *P*?

4 What is the distance from *O* vnto *P*? And why the way should be more from *L* vnto *P*, going by *O*, then by *N*?

5 What is the Rumb from *P* vnto *L*?

6 What is the distance from *P* vnto *L*?

7 What is the Rumb from *N* vnto *O*?

8 What is the distance from *N* vnto *O*? And why it should not be the like Rumb and distance from *N* vnto *O*, as from *P* vnto *L*?

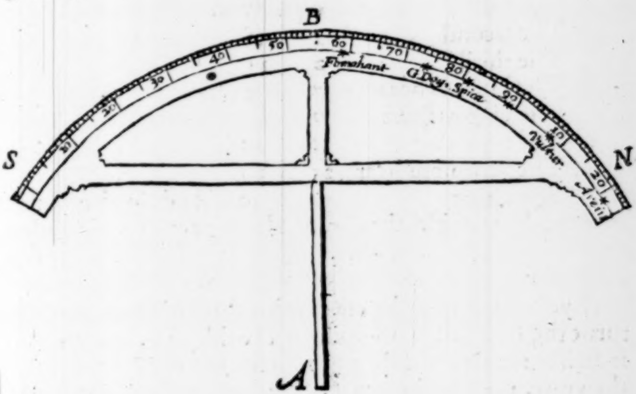
These questions well considered, and either resolued by the Staffe, or pricked downe on the chart, and compared with the globe and the common Sea-chart, will giue some light to the direction of a course, and reduction of places to their due longitude, which are now fouly distorted in the common Sea-charts.

An Appendix concerning

The description and vse of an instrument, made in forme of a Crosse-bow, *for the more easie finding of the latitude at Sea.*

THe former *Prop.* suppose the latitude to be knowne, I will here shew how it may be easily obserued.

Vpon the center *A*, and semidiameter *A B*, describe an ark of a circle *S B N*. The same semidiameter will set of *60 gr.* from *B* vnto *S* for the South end, and other *60 gr.* from *B* vnto *N* for the North end of the Bow: so the whole Bow will containe 120 *gr.* the third part of a circle. Let it therefore be diuided into so many degrees, and each degree subdiuided into six parts, that each part may be ten minutes: but let the numbers set to it be 5. 10. 15. vnto 90 *gr.* and then againe 5. 10. 15. vnto 25. that 55 may fall in the middle, as in this figure.



The Bow being thus diuided and numbred, you may see the moneths and dayes of each moneth vpon the backe, and such

ſuch ſtarres as are fit for obſeruation vpon the ſide of the Bow.

If you deſire to make vſe of it in North latitude, you may number 23 *gr.*30 *m.* from 90 towards the end of the Bowe at N, and there place the tenth day of Iune. And 23 *gr.*30 *m.* from 90 toward *S*; and there at 66 *gr.*30 *m.* place the tenth day of December. And ſo the reſt of the dayes of the yeare, according to the declination of the Sunne at the ſame dayes. The ſtarres may be placed in like maner according to their declinations.

Arcturus at	21 *gr.*	10 *m.*
The Buls eye	15	42
The Lions heart	13	45
The Vultures heart	7	58
The little dog	6	9

from 90 toward the North end of the Bow at N. Then for Southerne ſtarres, you may number their declination from 90 toward the South end of the Bow at *S*. As firſt the three ſtarres in *Orions* girdle,

The firſt at	0 *gr.*	37 *m.*
The ſecond	1	28
The third	2	11
The Hydra's heart	7	5
The virgins ſpike	9	10
The great dog	16	12
The Scorpions heart	25	30
Fomahant	31	30

And ſo the South crowne, the triangle, the clouds, the croſiers, or what other ſtarres you think fit for obſeruation. This I call the fore ſide of the Bow.

If you deſire to make vſe of it in South latitude, you may turne the Bow, and diuide the backe ſide of it, and number it in like maner; and then put on the moneths and dayes of the yeare, placing the tenth of December at the South end, and the tenth of Iune toward the middle of the Bow, and the reſt of the dayes according to the Sunnes declination as before.

The

The chiefest of the Northerne starres may here be placed in like maner according to their declination, Anno 1625.

The pole starre at		87 *gr.*	20 *m.*
The first guard		75	45
The second guard		73	25
The great Beares backe		63	45
In the great Beares taile	first	58	2
	second	57	55
	third	51	15
The side of Perseus		48	28
The goate		45	33
The taile of the swan		44	0
The head of Medusa		39	30
The harp		38	30
Castor		32	38
Pollux		28	52
The North crowne		28	0
The Rams head		21	40
Arcturus		21	10
The Buls eye		15	42
The Lions heart		13	45
The Vultures heart		7	58
Orions right shoulder		7	17
Orions left shoulder		5	57

And so any other starre, whose declination is knowne vnto you, which being done. The vse of this Bow may be

1 *The day of the moneth being knowne, to finde the declination of the Sunne.*

2 *The declination being giuen, to finde the day of the moneth.*

These two *Prop.* depend on the making of the Bow. If the day be knowne, looke it out in the backe of the Bow: so the declination will appeare in the side. Or if the declination be knowne, the day of the moneth is set ouer against it. As if the day of the moneth were the 14 of July: looke for

this day in the backe of the Bow, and you ſhall find it ouer a-gainſt 20 *gr.* of North declination. If the declination giuen be 20 *gr.* to the Southward, you ſhal find the day to be either the eleuenth of Nouember, or the eleuenth of Ianuary.

3 *To find the altitude of the Sunne or ſtarres.*

Here it is fit to haue two running ſights, which may be eaſily moued on the backe of the Bow. The vpper ſight may be ſet either to 60 *gr.* or to 70 *gr.* or to 80 *gr.* as you ſhall find to be moſt conuenient: the other ſight may be ſet on, to any place betweene the middle and the other end of the Bow. Then with the one hand hold the center of the Bow to your eye, ſo as you may ſee the Sunne or ſtarre by the vpper ſight, and with the other hand moue the lower ſight vp or downe vntill you haue brought one of the edges of it to be euen with the horizon (as when you obſerue with the Croſſe-ſtaffe:) ſo the degrees contained betweene that edge and the vpper ſight, ſhall ſhew the altitude required.

Thus if the vpper ſight ſhal be at 80 *gr.* and the lower ſight at 50 *gr.* the altitude required is 30 *gr.*

4 *To find any North latitude, by knowing either the day of the moneth, or the declination of the Sunne.*

As oft as you are to obſerue in North latitude, place both the ſights on the fore ſide of the Bow, the vpper ſight to the declination of the Sunne, or the day of the moneth at the North end, and the lower ſight toward the South end. Then when the Sunne cometh to the meridian, turne your face to the South, and with the one hand hold the center of the Bow to your eye, ſo as you may ſee the Sunne by the vpper ſight; with the other hand moue the lower ſight, vntill you haue brought one of the edges of it to be euen with the horizon: ſo that edge of the lower ſight ſhall ſhew the latitude of the place in the fore ſide of the Bow.

Thus being in North latitude vpon the ninth of October: if

if I set the vpper sight to this day, at the fore side and North end of the Bow, I shall find it to fall to the Southward of 90 vpon 80 *gr.* and therefore at 10 *gr.* of South declination. Then the Sunne coming to the meridian, I may set the center of the Bow to mine eye, as if I went to find the altitude of the Sunne, holding the North end of the Bow vpward, with the vpper sight betweene mine eye and the Sunne, and mouing the lower sight, vntill it come to be euen with the horizon. If here the lower sight shall stay at 50 *gr.* I may well say, that the latitude is 50 *gr.* For the meridian altitude of the Sunne is 30 *gr.* by the last *Prop.* and the Sun hauing 10 *gr.* of South declination, the meridian altitude of the equator would be 40 *gr*; and therefore the obseruation was made in 50 *gr.* of North latitude.

By the same reason, if the lower side had stayed at 51 *gr.* 30 *m.* the latitude must haue been 51 *gr.* 30 *m.* and so in the rest.

5 *To find any North latitude, by the meridian altitude of the starres to the Southward.*

Let the vpper sight be set to the starre, which you intend to obserue, here placed in the fore side of the Bow. Then hold the North end of the Bow vpward, and turning your face to the South, obserue the meridian altitude as before: so the lower sight shall shew the latitude of the place in the fore side of the Bow.

Thus if in obseruing the meridian altitude of the great Dog-starre, the lower sight shall stay at 50 *gr.* it would shew the latitude to be 50 *gr.* For this starre being here placed at 73 *gr.* 48 *m.* if we take thence 50 *gr.* his meridian altitude would be 23 *gr.* 48 *m.* to this if we adde 16 *gr.* 12 *m.* for the South declination of this starre, it would shew the meridian altitude of the equator to be 40 *gr.* and therefore the latitude to be 50 *gr.*

6 To find any North latitude, by the meridian altitude of the starres to the Northward.

Let the vpper sight be set to the starre which you intend to obserue, here placed on the backe side of the Bow. Then hold the North end of the Bow vpward, and turning your face to the North, obserue the altitude of the starre when he cometh to be in the meridian and vnder the pole: so the lower sight shall shew the altitude of the pole in the back side of the Bow.

Thus the former guard coming to be in the meridian vnder the pole, if you obserue and find the lower sight to stay at 50 *gr.* such is the eleuation of the pole, and the latitude of the place to the Northward. For the distance betweene the two sights will shew the altitude to be 35 *gr.* 45 *m.* & the star is 14 *gr.* 15 *m.* distant from the North pole. These two doe make vp 50 *gr.* for the eleuation of the North-pole, and therefore such is the North latitude.

7 To find any South latitude, by knowing either the day of the moneth, or the declination of the Sunne.

When you are come into South latitude, turne both your sights to the backside of the Bow: the vpper sight to the declination of the Sun, or the day of the moneth at the South end, and the lower sight toward the North end of the Bow. Then the Sunne coming to the meridian, turne your face to the North, and holding the South end of the Bow vpward, obserue the meridian altitude as before: so the lower sight shall shew the latitude of the place in the backe side of the Bow.

Thus being in South latitude, vpon the tenth of May if you obserue and finde the lower sight to stay at 30 *gr.* on the backe side of the Bow, such is the latitude. For the declination is 20 *gr.* Northward, the altitude of the Sunne betweene the two sights 40 *gr.* the altitude of the equator 60 *gr.* and there-

therefore the latitude 30 *gr.*

8 *To find any South latitude, by the meridian altitude of the starres to the Northward.*

Let the vpper sight be set to the starre which you intend to obserue, here placed on the backe side of the Bow. Then hold the South end of the Bow vpward, and turning your face to the North, obserue the meridian altitude as before: so the lower sight shall shew the latitude of the place in the back side of the Bow.

Thus being in South latitude, and the former guard comming to be in the meridian ouer the pole. If you obserue and finde the lower sight to stay at 5 *gr.* such is the latitude. For this starre is 14 *gr.* 15 *m.* from the North pole, the altitude of the starre betweene the two sights 9 *gr.* 15 *m.* the North pole depressed 5 *gr.* and therefore the latitude 5 *gr.* to the Southward.

9 *To obserue the altitude of the Sunne backward.*

Set the vpper sight either to 60, or 70, or 80 *gr.* as you shall find it to be most conuenient, the lower sight on any place betweene the middle and the other end of the Bow, and haue an horizontall sight to be set to the center. Then may you turne your backe to the Sunne, and the back of the Bow toward your selfe, looking by the lower sight through the horizontall sight, and mouing the lower sight vp & downe, vntill the vpper sight doe cast a shadow vpon the middle of the horizontall sight: so the degrees contained betweene the two sights on the Bow, shall giue the altitude required.

Thus if the vpper sight shall be at 80 *gr.* and the lower sight at 50 *gr.* the altitude required is 30 *gr.* as in the third *Prop.*

10 *To find any North latitude by a backe obseruation, knowing either the day of the moneth, or the declination of the Sunne.*

When you obserue in North latitude, place your three sights on the fore side of the Bow: the vpper sight to the declination of the Sun, or the day of the moneth, at the North end; the lower sight toward the South end of the *B*ow; and the horizontall sight to the center. Then the Sunne coming to the meridian, turne your face to the North, & holding the North end of the *B*ow vpward, the South end downeward, with the back of it toward your selfe, obserue the shadow of the vpper sight as in the former *Prop.* so the lower sight shall shew the latitude of the place in the fore side of the *B*ow.

Thus being in North latitude vpon the ninth of October, if you obserue and find the lower sight to stay at 50 *gr.* on the fore side of the Bow, such is the latitude. For the declination is 10 *gr.* Southward, and the altitude of the Sunne betweene the two sights 30 *gr.* the altitude of the equator 40 *gr.* and therefore the latitude 50 *gr.* as in the fourth *Prop.*

11 *To find any South latitude by a back obseruation, knowing either the day of the moneth, or the declination of the Sunne.*

When you obserue in South latitude, place your three sights on the backe side of the Bow: the vpper sight to the declination of the Sunne, or the day of the moneth at the South end; the lower sight toward the North end of the Bow, and the horizontall sight to the center. Then the Sun coming to the meridian, turne your face to the South, and holding the South end of the Bow vpward, with the backe of it toward your selfe, obserue the shadow of the vpper sight as before: so the lower sight shal shew the latitude of the place in the back side of the Bow.

Thus being in the South latitude vpon the tenth of May, if

if you obserue and find the lower sight to stay at 30 *gr.* on the backe of the Bow, such is the altitude. For the declination is 20 *gr.* Northward, the altitude of the Sunne betweene the two sights 40 *gr.* the altitude of the equator 60 *gr.* and therefore the latitude 30 *gr.* as in the seuenth *Prop.*

12 *To find the day of the moneth, by knowing the latitude of the place, and obseruing the meridian altitude of the Sunne.*

Place your three sights according to your latitude; the horizontall sight to the center, the lower sight to the latitude, and the vpper sight among the moneths. Then when the Sunne cometh to the meridian, obserue the altitude, looking by the lower sight through the horizontall, and keeping the lower sight still at the latitude, but mouing the vpper sight vntil it giue shadow vpon the middle of the horizontal sight: so the vpper sight shall shew the day of the moneth required.

Thus in our latitude if you set the lower sight to 51 *gr.* 30 *m.* and obseruing finde the altitude of the Sunne betweene that and the vpper sight to be 28 *gr.* 30 *m.* this vpper sight will fall vpon the ninth of October, and the twelfth of Februarie. And if yet you doubt which of them two is the day, you may expect another meridian altitude; and then if you find the vpper sight vpon the tenth of October, and the eleuenth of Februarie, the question will be soone resolued.

13 *To find the declination of any vnknowne starre, and so to place it on the* Bow, *by knowing the latitude of the place, and obseruing the Meridian altitude of the Starre.*

When you find a starre in the Meridian that is fit for obseruation. Set the center of the Bow to your eye, the lower sight to the latitude, and moue the vpper sight vp or downe vntill you see the horizon by the lower sight, and the starre

by

by the vpper sight, then will the vpper sight stay at the declination and place of the starre.

Thus being in 20 *gr.* of North latitude, if you obserue and find the meridian altitude of the head of the Crosier to be 14 *gr.* 50 *m.* The vpper sight will stay at 34 *gr.* 50 *m.* and there may you place this starre. For by this obseruatiō the distance of this starre from the South pole should be 34 *gr.* 50 *m.* and the declination from the equator 55 *gr.* 10 *m.* And so for the rest.

The starres which I mentioned before, do come to the meridian in this order, after the first point of *Aries.*

	Ho.	Mi.		Ho.	Mi.
The pole starre at	0	29	The lions hart	9	48
The rams head	1	46	The great beares backe	10	40
The head of Medusa	2	44	First in gr. beares taile	12	37
The side of Perseus	2	58	The Virgins spike	13	5
The Buls eye.	4	15	Second in gr. bea. taile	13	9
The goate	4	49	Third in gr. beares taile	13	33
Orions left shoulder	5	5	Arcturus	13	58
Orions girdle the first	5	13	The formost guard	14	52
Orions girdle the second	5	17	The North crowne	15	19
Orions girdle the third.	5	22	The hindmost guard	15	25
Orions right shoulder	5	35	Scorpions hart	16	7
The great dog	6	29	The harpe	18	24
Castor	7	10	Vulturs hart	19	33
The little dog	7	20	Swans taile	20	29
Pollux	7	22	Fomahant	22	36
The Hydra's hart	9	9			

The end of the second Booke.